A carbon cycle coupled climate model of Neoproterozoic glaciation: Influence of continental configuration on the formation of a "soft snowball"

Yonggang Liu[1] and W. Richard Peltier[1]

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[1] We examine the general conditions that must be satisfied by the configuration of the continents in order that steady state solutions for Neoproterozoic climate exist that are characterized by heavy continental glaciation but for which a substantial area of open water in the equatorial region persists. Such solutions have previously been termed "soft snowball" or "slush ball" to distinguish them from the "hard snowball" solutions that some have suggested be required to fit the observational constraints. It is found that three conditions are critical in this regard: (1) the continental area in high latitudes should be large enough that a massive ice sheet may develop even when pCO_2 is relatively high, an ice sheet complex that is subsequently capable of flowing to lower latitude. (2) The continental fragments in low latitude must be connected (or separated only by continental shelves above which water depths are small) to a significant degree with those at higher latitudes. (3) A relatively simple supercontinental outline favors the formation of the "soft snowball" state. Although the latter requirement is important, we have nevertheless found that soft snowball solutions are realized for the realistic Sturtian continental configuration of Li et al. (2008) that existed at ~720 Ma. However, in order for these states to exist, the positions of the individual continental fragments must be slightly adjusted so as to improve their connectivity. These adjustments are consistent with the error bars on the paleomagnetic inferences of paleolatitude and the even less well constrained paleolongitude. We also demonstrate that "soft snowball" solutions do not exist in models devoid of active continental ice sheets capable of flowing over the landscape. These results for Sturtian conditions extend our previously published results for the Marinoan period during which the supercontinent was centered upon much higher latitudes.

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1. Introduction

[2] The discovery of the existence of low-latitude glacial deposits at sea level [e.g., *Harland and Bidgood*, 1959; *Kirschvink*, 1992; *Hoffman and Schrag*, 2002, and references therein] during the Neoproterozoic Era (1000–542 Ma) on the basis of sedimentological evidence from many of the present-day continents has led to the reemergence of the idea of global glaciation, "die Eiszeit," a notion initially suggested by *Agassiz* [1840]. The central issue in this continuing debate is whether or not glaciation was in fact complete, with the continents fully covered by thick ice sheets and the oceans by sea ice.

[3] Advocates of this "hard snowball" Earth hypothesis, so named by *Kirschvink* [1992] and championed by *Hoffman et al.* [1998] and *Hoffman and Schrag* [2000, 2002], have suggested that the Earth could have entered such a "hard snowball" regime as a consequence of a runaway ice-albedo effect, a regime that would have lasted for tens of millions of years, from which later recovery would have occurred owing to the accumulation of atmospheric greenhouse gases (mainly CO_2) due to volcanic outgassing. During such extended glacial periods, photosynthetic activity would have essentially ceased owing to the existence of thick ice over both continents and ocean. This would have led to a shift of $\delta^{13}C$ of the inorganic carbon reservoir of the ocean toward more negative values, such as are clearly recorded in the "cap" carbonates deposited at the termination of the Neoproterozoic glacial intervals. Other natural consequences of the existence of complete ice cover would include deep ocean anoxia, an outcome of which would be the occurrence of banded iron formations [*Kirschvink*, 1992], and an essentially absent hydrological cycle [*Hoffman et al.*, 1998].

[4] On the other side of this debate are advocates of the so-called "soft snowball" Earth alternative hypothesis, first suggested as plausible in the work of *Hyde et al.* [2000], and a "high obliquity" hypothesis (see *Williams* [2008] for a

[1]Department of Physics, University of Toronto, Toronto, Ontario, Canada.

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review). The former is simply a less severe version of the snowball Earth hypothesis. Rather than the entire Earth being covered by ice, only the continents and the high-latitude oceans are entirely glaciated, the equatorial ocean remaining sea ice free over a substantial area. However, this modest difference in ice cover over the oceans has distinctly different implications, both for biological evolution and for the functioning of the hydrological cycle. In the soft snowball regime, primary production in the ocean remains active, and the hydrological cycle continues to impact the continents, in part owing to the dynamical activity of continental ice sheets [e.g., *Etienne et al.*, 2007; *Allen and Etienne*, 2008]. The main issue that has been raised concerning the plausibility of this alternative view [*Schrag and Hoffman*, 2001] is that a "soft snowball" might not be maintainable for many tens of millions of years that some have argued to be required by the geological evidence. The timescale of the Neoproterozoic Sturtian glaciation has probably been most accurately quantified by counting the number of geomagnetic reversals that are found within the stratigraphy of the glaciogenic deposits. This method delivers an estimate of 1–10 million years [*Sohl et al.*, 1999]; the lower end of this range of estimates may be the more plausible [*Raub*, 2008]. The duration of a "soft snowball" glaciation is entirely consistent with this estimated range [*Hyde et al.*, 2001; *Peltier et al.*, 2007]. It has also been suggested that the progressive retreat of ice in the "soft snowball" scenario may be incapable of producing the sudden precipitation of cap carbonates [*Schrag and Hoffman*, 2001]. As shown by *Crowley et al.* [2001] and *Peltier et al.* [2007], however, the final retreat of both land ice and sea ice in the "soft snowball" scenario occurs when a certain threshold of pCO_2 is crossed and occurs rapidly [see also *Hyde et al.*, 2001].

[5] The high-obliquity hypothesis, on the other hand, interprets the low-latitude glacial deposits to arise as a consequence of extremely high obliquity (>53°) of the Earth's spin axis with respect to the plane of the ecliptic. In such a high-obliquity state, the equatorial region would be colder than the polar region, and the existence of low-latitude ice cover need not imply global glaciation. This hypothesis suffers from the absence of a plausible mechanism that could explain why the obliquity would thereafter have decreased to the modern value of approximately 23.5 degrees. This idea has also been challenged by the evaporite evidence [e.g., *Kirschvink*, 1992, and references therein], which suggests that the obliquity of the Earth has remained low during the past two billion years [*Evans*, 2006]. The hypothesis is also undermined on the basis of numerical modeling [*Chandler and Sohl*, 2000; *Donnadieu et al.*, 2002]. However, the evidence for a strong seasonal cycle in the tropical region [*Williams*, 2008], as might be taken to be suggested by the sand wedges found in the sediments still need to be explained if this hypothesis is to be rejected completely. A more accurate way of estimating the obliquity of the spin axis during the Proterozoic Eon should be developed as the paleolatitude of the evaporites are not precisely constrained [*Williams*, 2008].

[6] It is nevertheless our view that the high-obliquity hypothesis is the least likely of the explanations of low-latitude glaciations. The "hard snowball" hypothesis has also been found wanting in a number of recent studies. For example, from an observational perspective, the existence of a near shutdown of the hydrological cycle, a necessity of "hard snowball" conditions, has been questioned by *Christie-Blick et al.* [1999]. More recently, compelling evidence [*Leather et al.*, 2002; *Etienne et al.*, 2007; *Rieu et al.*, 2007; *Allen and Etienne*, 2008] has been found in the sedimentology of the Huqf supergroup, Oman, demonstrating that the hydrological cycling continued to operate during a primary Neoproterozoic glaciation event. If continuous operation of the hydrological cycle can be further demonstrated to be global and synglacial on the basis of more precise age measurements, this would entirely undermine the "hard snowball" hypothesis. Furthermore, if photosynthetic life had been essentially eliminated as is likely implied by this hypothesis, the occurrence of the subsequent Cambrian Explosion following the Cryogenian and Ediacaran periods, in which Eukaryotic life proliferated would be hard to comprehend. In fact, biomarker evidence found in southeastern Brazil [*Olcott et al.*, 2005] reveals that primary production and photosynthesis in the ocean remained strong during the glacial events. Furthermore, the diversity of microbials was found to be nearly invariant through the glacial events recorded in the sediments in both Death Valley, USA, and Australia [*Corsetti et al.*, 2003, 2006]. This list of surviving photosynthetic biota through the glaciations is increasing [e.g., *Moczydlowska*, 2008; *Love et al.*, 2009]. In fact, the work by *Nagy et al.* [2009] clearly demonstrates that the biological turnover occurred ~16 Myr [*Corsetti*, 2009] before the earliest Neoproterozoic global glaciation, the Sturtian glaciation, invalidating the causal link between the two events. This does not deny the occurrence of a "hard snowball," but does imply that the biological turnover may not serve as supportive evidence of the "hard snowball" hypothesis. It should be noted that in the "hard snowball" hypothesis, refugia could exist [e.g., *Hoffman and Schrag*, 2002] in which primitive life could survive and might proliferate. Therefore, not until further evidence is discovered demonstrating that biological activity existed globally and synglacially, can the "hard snowball" hypothesis be simply refuted by appealing to such activity. Nonetheless, these data potentially suggest that sea ice cover was not complete during the glacial periods.

[7] From a modeling perspective, numerical simulation by *Poulsen* [2003] has shown that the occurrence of a runaway ice-albedo effect might not be as straight forward as speculated on the basis of simple energy balance models, there being many additional factors that must be taken into account, as further discussed by *Poulsen and Jacob* [2004]. The thin ice solution which was proposed [*McKay*, 2000; *Pollard and Kasting*, 2005] as one of the possible ways for the photosynthetic processes to continue during a "hard snowball" event has also been demonstrated to be unlikely [*Warren et al.*, 2002; *Warren and Brandt*, 2006]. Further discussion is provided by *Allen* [2006]. A more recent issue that has been raised concerning the "hard snowball" hypothesis is the difficulty in triggering the final deglaciation. It has been suggested by numerical modeling that > 0.29 bar of atmospheric pCO_2 needs to be accumulated to melt the hard snowball [*Pierrehumbert*, 2004, 2005]. It has been assumed that there would be no gas exchange between the atmosphere and the ocean, so that 0.29 bar of CO_2 can be accumulated in ~12 Myr if the same volcanic outgassing rate (~5.4×10^{12} mole yr^{-1}) as is occurring at present is assumed [*Hoffman et al.*, 1998]. However, small areas of open waters could exist even in a "hard snowball" owing to lava flow or other hydrothermal activities [*Le Hir et al.*, 2008], which could lead to efficient

gas exchange between the atmosphere and the ocean if the area were larger than ~3 × 10^3 km^2. If this process were operative, it would take longer than 30 Myr to accumulate 0.29 bar of CO_2 even with an outgassing rate of ~8.6 × 10^{12} mole yr^{-1} [*Le Hir et al.*, 2008], which is much longer than the nominal value of ~10 Myr for the duration of the hard snowball [*Hoffman et al.*, 1998; *Hoffman and Schrag*, 2000]. On the basis of new age measurements, the duration of the Marinoan glaciation is constrained to be at most 23.5 Myr [*Hoffman and Li*, 2009], and that of Sturtian glaciation could be as short as 5 Myr [*Macdonald et al.*, 2010]. In order to escape from this requirement for extremely high pCO_2, it has recently been argued that the surface albedo of the hard snowball could be significantly lowered in the tropics owing to the accumulation of dust in the net tropical ablation zone that is characteristic of the atmosphere when the Earth is completely ice covered [*Abbot and Pierrehumbert*, 2010]. The major sources of the dust envisioned by *Abbot and Pierrehumbert* [2010] include that blown by wind from unglaciated land and that due to volcanic eruptions. Neither of these two sources seems to be entirely realistic as the area of exposed land which would be available for dust production would be expected to be small (e.g., consider Antarctica and Greenland at present), and volcanic eruptions should produce much less dust than under present-day conditions since the land is envisioned as being almost entirely covered by thick ice sheets under hard snowball conditions. Nevertheless, the "hard snowball" hypothesis may not be entirely ruled out as yet on the basis of the few (critical but perhaps local) observations, nor by the existing numerical simulations which themselves have limitations and many uncertainties in the values selected for their governing parameters. Further multidisciplinary work is surely required to obtain a definitive conclusion concerning the plausibility of the "hard snowball" model.

[8] The alternative soft snowball model of Neoproterozoic glaciation has been suggested [*Hyde et al.*, 2000; *Peltier et al.*, 2004] to be stable at near modern atmospheric CO_2 concentrations (pCO_2) even when the solar insolation is taken to be ~7% weaker during the late Neoproterozoic period. However, if pCO_2 continued to decrease owing to intensified weathering associated with the breakup of the supercontinent of Rodinia [e.g., *Donnadieu et al.*, 2004], the planet might not have been able to escape from a hard snowball "fate" if there were no negative feedback mechanism acting that could stabilize this trend, assuming that the glaciation was indeed caused by decreased pCO_2, and not by other processes [e.g., *Schrag et al.*, 2002; *Pavlov et al.*, 2005]. *Peltier et al.* [2007] proposed that enhanced remineralization of the dissolved organic carbon (DOC) reservoir at low temperature could have played the role of such a negative feedback. This idea was based on the hypothesis that an extraordinarily massive DOC reservoir existed in the Neoproterozoic ocean [*Rothman et al.*, 2003]. When the temperature at the Earth's surface decreases substantially, the solubility of O_2 in seawater increases significantly [*García and Gordon*, 1992], which in turn increases the rate of oxidization/remineralization of the DOC into dissolved inorganic carbon (DIC = CO_2). A portion of this dissolved CO_2 in the ocean would be partitioned into the atmosphere, and in this way an initial climate cooling may be not only stabilized but actually reversed. The unique nature of the Neoproterozoic carbon cycle may therefore have acted to prevent the occurrence of a "hard snowball" Earth. An equally interesting aspect of the solutions to the Neoproterozoic climate model described by *Peltier et al.* [2007] concerns its ability to deliver agreement with the inferred negative excursions of $\delta^{13}C$ as a consequence of nonequilibrium carbon cycling, owing to the exceptionally large size and long residence time of organic carbon relative to that of inorganic carbon in the Neoproterozoic ocean. The model of *Peltier et al.* [2007] therefore provides an alternative to the hard snowball hypothesis of *Hoffman et al.* [1998], one in which a globally active hydrological cycle continues to function.

[9] However, it was clearly demonstrated by *Peltier et al.* [2007] that the behavior of the carbon cycle coupled climate model is tightly constrained by the nature of the stand-alone climate model, for example, the hysteresis loop of steady state solutions of the model in mean sea level temperature-pCO_2 space that exists in the absence of the coupling to the carbon cycle which is the source of model time dependence. In particular, the existence and the stability of oasis (soft snowball) states is crucial in order for the carbon cycle coupled model to be able to reverse an initial cooling trend. Since the latitudinal distribution of continents has been considered critically important to the success of the "hard snowball" Earth hypothesis [*Hoffman and Schrag*, 2000, 2002] and since the "soft snowball" alternative has been advanced in terms of a continental configuration of Marinoan type for which equatorial continentality is low and therefore at odds with that characteristic of the initial Sturtian event, it is clearly necessary to investigate the extent to which a shift to a more Sturtian paleogeography might eliminate the "soft snowball" branch of solutions. The paleogeography appropriate for the Sturtian event (the first of the three potentially global glaciations that occurred during Neoproterozoic time [see *Peltier et al.*, 2007, Figure 1]) was significantly different from that appropriate for the subsequent Marinoan event [*Li et al.*, 2008] (compare Figures 1a and 1b and see also *Trindade and Macouin* [2007]) as employed by *Peltier et al.* [2007]. Our purpose herein is to provide a detailed assessment of the sensitivity of the existence of oasis solutions to extreme variations of the paleocontinental configuration. The associated question of the stability of the oasis states, when subjected to perturbations (e.g., variations of solar constant or pCO_2), and the influence on the evolution of the carbon cycle coupled version of the model is investigated in a separate paper (Y. Liu and W. R. Peltier, A coupled carbon cycle-climate model of Neoproterozoic glaciation: Explicit carbon cycle with stochastic perturbations, submitted to *Journal of Geophysical Research*, 2010). A further question raised by *Goddéris and Donnadieu* [2008] (see the reply by *Peltier and Liu* [2008]) concerning the question of the amount of CO_2 that would be produced by the action of the negative feedback proposed by *Peltier et al.* [2007] is also addressed in our other paper (Liu and Peltier, submitted, 2010). Here our concern will be exclusively with the "continentality complaint" of *Hoffman et al.* [2008].

[10] In the analyses to follow, two distinctly different representations of the continental configuration will be considered, one of which consists of a single supercontinental mass of elliptical form whose mean latitude is considered to be a control variable of the model. The second supercontinental model will consist of the realistic 720 Ma continental configuration based on the reconstruction of *Li et al.* [2008]. Experiments with this more realistic model will also

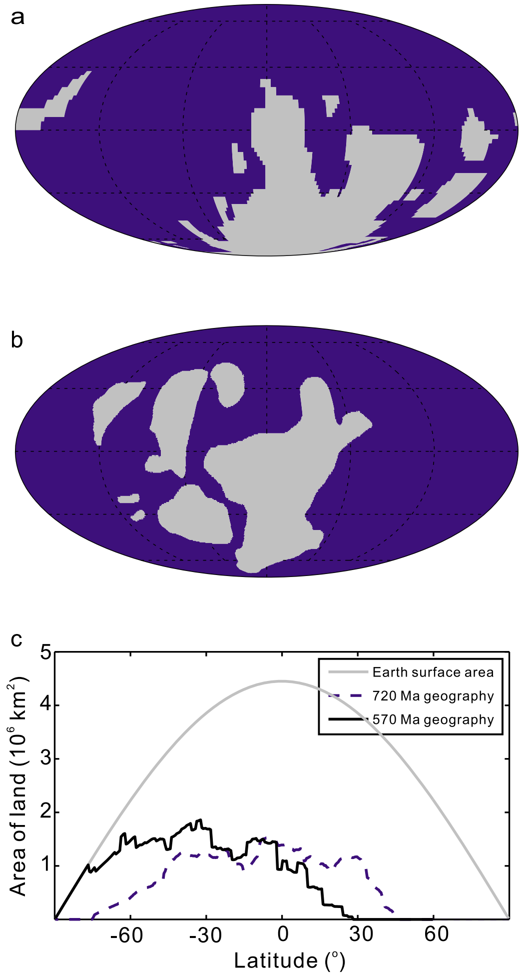

Figure 1. (a) Paleogeography used by *Peltier et al.* [2007], which was based on the reconstruction by *Dalziel* [1997] appropriate for 570 Ma. (b) Paleogeography for 720 Ma [*Li et al.*, 2008]. (c) Land area distribution with latitude for the geography in Figure 1a (black dashed line) and Figure 1b (black solid line). The Earth surface area is also shown (gray line) so that the distribution of ocean area with latitude is conveniently inferred from the space between the black dashed line or black solid line and the gray line.

include allowable (by the paleomagnetic constraints) shifts in latitudinal position of the centroid of continental mass as well as an investigation of the degree of connectivity between the individual fragments of which the supercontinent is composed.

[11] In sections 2 and 3 the properties of the ice sheet coupled energy balance climate model to be employed are summarized and detailed descriptions of the paleogeographies are provided, respectively. The results of the analyses we have performed with the model as a function of continental configuration are presented and discussed in section 4. Conclusions that follow from these analyses are provided in section 5.

2. Model Description

[12] The model to be employed herein consists of two primary components: A global surface energy balance module and a 3-D thermomechanical ice sheet module. The model also includes a thermodynamic sea ice module. The EBM and ice sheet model components have been described in detail in a series of papers [e.g., *Deblonde and Peltier*, 1991, 1993; *Tarasov and Peltier*, 1997, 2004], but we will provide a brief recapitulation of their most important characteristics for completeness’ sake and in order to clarify the particular assumptions that are made for the purpose of Neoproterozoic climate simulation.

2.1. Global Energy Balance Model

[13] The 2-D energy balance model (EBM) in this paper was originally developed by *North et al.* [1983] and then generalized somewhat by *Hyde et al.* [1989]. In this model the temperature at the surface of the Earth is determined by a solution to the following partial differential equation:

$$C(\mathbf{r})\frac{\partial T(\mathbf{r},t)}{\partial t} - \nabla_h \cdot [D(\theta)\nabla_h T(\mathbf{r},t)] + A + BT(\mathbf{r},t) = \frac{Q}{4}a(\mathbf{r},t)S(\theta,t), \quad \mathbf{r} = (\theta,\varphi). \tag{1}$$

In equation (1), θ is colatitude and φ is longitude of a surface point, $T(\mathbf{r}, t)$ is the sea surface temperature in degrees Celsius, and $C(\mathbf{r})$ is the effective surface heat capacity. This space-dependent surface heat capacity is employed to distinguish land, ocean, land ice and sea ice from one another for which we will assume the respective values 0.08, 9.7, 9.7/60 and 0.97. The latitude-dependent parameter D is a thermal diffusivity that is employed to represent the combined influence of all forms of heat transport including the poleward transport of heat due to the combined influence of the Hadley circulation and midlatitude baroclinic waves. The term $A+BT(\mathbf{r}, t)$ represents the outgoing infrared radiation (IR) from the top of atmosphere in terms of the surface temperature, in which A and B are empirical constants taken to have values 203.3 W m^{-2} and 2.10 W m^{-2} K^{-1}, respectively [*North et al.*, 1981], on the basis of fits to modern climate observations with $p\mathrm{CO}_2$ approximately equal to 300 ppmv. Q is the solar constant, the modern value of which is 1360 W m^{-2}, but this must be decreased by 6–7% during the late Neoproterozoic period to account for the fact that main sequence stars such as our Sun burn more brightly as they age [*Gough*, 1981]. The a is the coalbedo of the surface of the Earth which is assumed to be a smooth function of latitude only, in which the land and ocean are not distinguished [see *Deblonde and Peltier*, 1991] if there is no snow, land ice or sea ice present. S is the solar distribution function which varies strongly with latitude and orbital geometry. Orbital forcing will not be considered for the purpose of this paper since Neoproterozoic orbital parameters are unknown; however, this influence will be considered elsewhere (Liu and Peltier, submitted, 2010).

[14] The seasonal change of snow cover on land and sea ice cover on the ocean, are resolved in the model. Sea ice is assumed to form when the temperature of the ocean is below −4°C. Since it is purely thermodynamic, the thickness and the flow of sea ice are not considered in the model. Because they are sensitive to temperature but in turn affect the temperature field through modification of the surface coalbedo ($a(\mathbf{r}, t)$) of the planet, equation (1) becomes highly nonlinear and must be solved iteratively in order to obtain a converged solution. Land ice cover changes slowly with time, and thus is con-

sidered to be constant during the course of a single year. A value of 0.3 is used for the coalbedo of both snow and land ice, and 0.55 for sea ice coalbedo. The argument for using a value for sea ice coalbedo of this order was provided by *Crowley et al.* [2001]. This is especially appropriate for low-latitude sea ice such as is crucial for the occurrence of "hard snowball" conditions as sea ice forming near the equator would be expected to be covered with melt ponds, thus lowering the albedo.

[15] The impact of the variation of CO_2 concentration on the radiative forcing at the surface may be introduced into the model through a modification of the variable A in equation (1) as follows:

$$d\mathrm{Rad} = k \ln\left(\frac{pCO_2}{pCO_{2,e}}\right), \tag{2}$$

where dRad is the increment in surface radiative forcing due to an increase in CO_2 concentration from the reference level $p\mathrm{CO}_{2,\,e}$, which is 300 ppmv here. The k is a constant whose value may be obtained through radiative-convective modeling utilizing the known spectral absorption characteristics of CO_2 [*Ramanathan et al.*, 1979; *Myhre and Stordal*, 1997; *Myhre et al.*, 1998] for which a value of 6.0 W m^{-2} is assumed [*Tarasov and Peltier*, 1997, 1999].

2.2. Thermomechanical Ice Sheet Model

[16] The 3-D thermomechanically coupled ice sheet model employed here is the University of Toronto Glacial Systems Model (UofT GSM), which is a further extension of that employed by *Hyde et al.* [2000], in which an isothermal version of the ice sheet model originally developed by *Deblonde et al.* [1992] was employed. The complete thermomechanical version of the model to be employed herein has been tested in detail in the context of the European EISMINT program [*Payne et al.*, 2000; *Tarasov and Peltier*, 1999].

[17] On the basis of the shallow ice approximation [*Paterson*, 1994], only vertical thermal diffusion and horizontal advection in the ice sheet are considered, as represented in the following partial differential equation:

$$\rho_i C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left\{k(T)\frac{dT}{dz}\right\} - \rho_i C(T)\mathbf{V}\cdot\nabla T + Q_d. \tag{3}$$

The values employed for the density of ice ρ_i, thermal diffusivity $k(T)$ and heat capacity $C(T)$ are the same as those by *Tarasov and Peltier* [1999], as are the boundary conditions. In equation (3), $\mathbf{V}$ is the horizontal flow velocity vector of ice and Q_d represents the heat production due to ice deformational heating or basal sliding. Only vertical diffusion is considered in the bedrock thermodynamic calculation, which is represented as

$$\rho_b C_b \frac{\partial T}{\partial t} = k_b \frac{\partial^2 T_b}{\partial z^2}, \tag{4}$$

in which the density, heat capacity, and thermal conductivity of the bedrock are set to the values 3300 kg m^{-3}, 1000 kJ kg^{-1} K^{-1}, and 3 W m^{-1} K^{-1}, respectively. The upper boundary condition for the determination of temperature in the bedrock is fixed by the temperature at the surface delivered by the EBM if no ice is present or by the temperature at the base of the ice sheet if the surface is ice covered. The lower boundary condition (2 km below the surface) is specified to be the heat flux due to the cooling of the Earth. A uniform value of 60 mW m^{-2} is employed for the purpose of the analyses to be presented herein, which is approximately the average continental heat flux appropriate for Neoproterozoic conditions [e.g., *Stein*, 1995; *Pari and Peltier*, 1995, 1998].

[18] The velocity of ice flow under its own weight, $\mathbf{V}$ in equation (3), is calculated using the standard Glen flow law [*Paterson*, 1994] under the shallow ice approximation:

$$\mathbf{V}(\mathbf{r}) = \mathbf{V}_b - 2(\rho_i g)^n\{\nabla_\mathrm{h}(h)\cdot\nabla_\mathrm{h}(h)\}^{(n-1)/2} \cdot \nabla_\mathrm{h}(h) \times E\int_{z_b}^{z} A\left(T^*(z')\right)(h-z')^n dz', \tag{5}$$

where $\mathbf{V}_b$ is the basal sliding velocity, h is the surface elevation relative to sea level, n is the flow law exponent (= 3 in the case that the Glen flow law is assumed), E is the flow enhancement factor (= 6 here, 6.5 was employed to fit the deglacial history of the Laurentide ice sheet that existed at Last Glacial Maximum approximately 21,000 years ago [*Tarasov and Peltier*, 2004]), $A(T^*)$ is the temperature-dependent flow coefficient in which T^* is the temperature of the ice in degrees Kelvin corrected for the pressure melting point [see *Tarasov and Peltier*, 1999, 2002]. Note that the subscript "h" on the gradient operator stands for "horizontal." The basal velocity is determined independently using an empirical relationship, as a function of the basal shear stress and basal temperature [e.g., *Tarasov and Peltier*, 2002, equations (6) and (7)]. The enhancement of basal sliding due to till deformation has been considered in modeling North American ice sheet history [*Tarasov and Peltier*, 2004], but is not considered in the version of the model to be employed here owing to the lack of information concerning Neoproterozoic till.

[19] Finally, ice thickness is computed through application of the mass conservation relation:

$$\frac{\partial H(x,y)}{\partial t} = -\nabla_\mathrm{h}\cdot\int_{z_b}^{h}\mathbf{V}(\mathbf{r})dz + G(\mathbf{r}, T(\mathbf{r})), \tag{6}$$

in which G is the net surface and basal mass balance, namely, the difference between the snow/ice accumulation at the surface and the ablation of snow/ice both at the surface and at the base [see *Tarasov and Peltier*, 1999, 2002]. A spatially uniform annual precipitation of 0.6 m is assumed at sea level, but the monthly precipitation obtained is reduced/increased locally by a factor of 1.03 for each degree by which the monthly mean temperature at sea level is lower/higher than 0°C. The precipitation change due to the desert/elevation effect [*Deblonde and Peltier*, 1993] is also included in the analysis of mass balance. The snow accumulation and melting are computed using the same formulation as that of *Tarasov and Peltier* [2002]. The influence of ice calving remains a significant problem in ice sheet modeling [*Benn et al.*, 2007]. For present purposes we will simply assume that calving is complete when the ice sheet advances from a continental region into the ocean.

[20] Bedrock deformation beneath the ice sheet is also incorporated into the model in order to better simulate the

evolving thickness of the ice sheet and is assumed to be adequately represented by the assumption of a simple damped return to equilibrium, as follows:

$$\frac{dh'}{dt} = -\frac{h' - h_0}{\tau} + \frac{\rho_i H + \rho_w \delta H_w}{\rho_e \tau}, \quad (7)$$

a formulation which provides an adequate approximation to the more realistic gravitationally self consistent isostatic adjustment theory for the viscoelastic mantle in response to a variation of surface mass load (e.g., see *Peltier* [1998, 2007] for recent reviews). In equation (7), h'_0 is the depression (elevation) of the bedrock surface relative to sea level in equilibrium under ice-free Neoproterozoic conditions, and h' is the depression induced by the evolving surface mass load associated with glaciation, ρ_i, ρ_w, and ρ_e are the average densities of ice, seawater, and lithosphere, respectively, H is the thickness of ice, δH_w is the change in the thickness of a water column, and τ is a single relaxation time constant (= 4 kyr) chosen so as to reasonably fit Late Quaternary postglacial rebound data [*Peltier*, 1998]. The viscoelastic structure of the Neoproterozoic mantle was probably similar to present, since the temperature of the interior has not changed significantly since Neoproterozoic time according to modern analyses of planetary thermal history [*Butler et al.*, 2005]. For present purposes we will not calculate sea level history explicitly so that δH_w is set to zero. The ice sheet model is solved at a resolution of 0.5° × 0.5° horizontally with 35 vertical levels.

2.3. Method for Calculating the Hysteresis Loop

[21] The hysteresis loop consists of a subset of the complete set of steady state solutions in mean sea level temperature-$p\mathrm{CO}_2$ space. In principle determination of the complete set of steady state solutions requires that we run the time-dependent model to steady state for each $p\mathrm{CO}_2$ value in the selected range. The annually averaged global mean sea level temperature so determined for each steady state defines one point in temperature-$p\mathrm{CO}_2$ space. Depending on the initial state in which the time-dependent model is initiated, multiple steady states may be found to exist for each specific $p\mathrm{CO}_2$ value. A hysteresis loop consists of a closed orbit in state space that includes such multiple equilibria. However, calculating the complete set of steady states in this way would require unnecessarily excessive computation. We instead adopt an approximate method since we are most interested in the existence of the "soft snowball"/oasis solutions, rather than the precise shape of the hysteresis loop. This method is the same as that employed by *Peltier et al.* [2004, 2007] but has not previously been adequately described. In this method, the model is initialized from a state in which continental glaciation is essentially absent (very little land ice or sea ice present) for which the $p\mathrm{CO}_2$ is high, for example, 1000 ppmv. Instead of running to a perfectly steady state for this carbon dioxide concentration, $p\mathrm{CO}_2$ is continuously but slowly decreased, for example (in terms of radiative forcing), at a pace 1 W m^{-2}/50 kyr, until the "hard snowball" state is eventually reached. If the climate passes through a "soft snowball" intermediate state during the process, then we initialize a new time-dependent calculation from this state during which $p\mathrm{CO}_2$ is increased slowly until the climate eventually returns to a hot ice-free state. Therefore, only approximate steady states are reached for each value of $p\mathrm{CO}_2$. If the pace of varying $p\mathrm{CO}_2$ is very slow, the approximate steady state solutions so obtained are found to be very close to precise equilibria. This is a consequence of the fact that, when $p\mathrm{CO}_2$ is high, very little land ice is present and the climate system reaches an approximate steady state very quickly (i.e., in one EBM time step of 500 years). Then by decreasing $p\mathrm{CO}_2$ slowly, the climate system will achieve a new approximate steady state in each time step because there will be little change in surface ice from that in the previous time step, the evolution of surface ice cover being the process that determines the rate of convergence. A pace of $p\mathrm{CO}_2$ reduction equivalent to a change in infrared forcing of 1 W m^{-2}/20 kyr or 1 W m^{-2}/50 kyr was usually employed in practice for our previous Marinoan integrations [*Peltier et al.*, 2004, 2007]. To determine the critical value for $p\mathrm{CO}_2$ at which the climate enters a "soft snowball" or "hard snowball" state suddenly from a hot state, the pace was sometimes decreased to 1 W m^{-2}/100 kyr. In the limit that the pace of CO_2 reduction tends to zero, this method delivers the same results as the exact method.

[22] From our experience with the model for the Marinoan glaciation, the pace of 1 W m^{-2}/100 kyr was initially considered slow enough and was initially employed to construct the set of steady state solutions for the Sturtian glaciation. This led us to reach an incorrect preliminary conclusion that "soft snowball" solutions did not exist for the 720 Ma continental configuration unless the supercontinent of Rodinia was shifted poleward by as much as 10°. We later discovered that for the more equatorially centered supercontinent the pace of $p\mathrm{CO}_2$ reduction was insufficiently slow to accurately determine the steady states. The reason for this was found to be related to the detailed continental configuration as is discussed in greater detail in section 4. Although this method of defining the set of steady state solutions therefore requires explicit testing to demonstrate that the results are robust against decreases in the rate of $p\mathrm{CO}_2$ reduction and increase, its merit is that it enables us to calculate continuous variations of land ice volume and sea ice coverage for continuous variations of $p\mathrm{CO}_2$. This greatly facilitates our understanding of the development of "soft snowball" states as we will demonstrate in sections 3 and 4. For present purposes we therefore choose to use a combined method. The approximate method is employed to sweep the whole $p\mathrm{CO}_2$ range with a choice of 1 W m^{-2}/100 kyr for the pace at which $p\mathrm{CO}_2$ is decreased, but a much slower pace of 1 W m^{-2}/800 kyr in the vicinity of possible "soft snowball" transition points (where climate shifts from the hot state to an oasis state). If "soft snowball" solutions are found, then no further verification calculations are needed, otherwise we perform several separate calculations at $p\mathrm{CO}_2$ values that are slightly higher than the "hard snowball" transition value (at which climate transits from a relative warm state into a "hard snowball" state). These calculations are each run to steady state to check that the solutions obtained using the approximate method were acceptably close to these solutions. If they are close, or at least on the same branch of the hysteresis loop, this implies that the oasis state truly does not exist. In this way we are able to combine the advantages of both the approximate method (rapidity of computation) and the exact method (accuracy).

[23] The method described above might lead the reader to the false conclusion that we believe that the $p\mathrm{CO}_2$ variation in

nature is necessarily slow. It needs to be kept in mind that we are here describing the state space of steady state solutions of an ice sheet coupled climate model that is NOT explicitly coupled to a model of the carbon cycle. The rate at which the carbon dioxide concentration of the atmosphere changes is actually determined by the collective effects of those physical processes that control the evolution of the carbon cycle itself and the way in which these processes are coupled to climate. The methodology we are employing to determine the complete set of model steady state solutions nevertheless does allow some insight into how one might enter into a hard snowball state. If the rate of decrease of atmospheric carbon dioxide concentration were sufficiently fast then insufficient time may be available for the oasis solution to develop.

[24] *Peltier et al.* [2007] demonstrate that a plausible negative feedback existed that could entirely prevent such a rate of decrease from occurring This feedback and the processes with which it would have been obliged to compete are the focus of the further analyses to be discussed elsewhere (Liu and Peltier, submitted, 2010).

3. Paleogeographic Reconstructions

[25] We will begin our analyses by discussing two geometrically simple supercontinental models with total land areas of ~105 × 10^6 km^2 (Figure 2) and ~130 × 10^6 km^2 (Figure 3), respectively. The shapes of these supercontinent models will be taken to be elliptical and to be placed either centered on the equator (Figure 2a), or displaced southward by 10° increments until the continent is centered on the South Pole (as partially illustrated in Figures 2b–2f). For each of these new positions of the supercontinent, we calculate the steady states of the climate model in the absence of coupling to the carbon cycle. The steady states are calculated for a range of values of atmospheric pCO_2 in order to investigate the nature of the hysteresis, if any, that may exist in the state space of mean surface temperature and atmospheric pCO_2. The methodology employed to compute these solutions is that discussed previously in section 2.3. By studying the nature of the solution space for these two variants upon continental geometry, we hope to gain insight concerning the influence of both latitudinal position and total area of the supercontinent upon ice sheet coupled climate model characteristics. In assessing the implications of these analyses, we will be primarily interested in the portion of the hysteresis loop that contains the multiple equilibria associated with the oasis branch and the hot branch of solutions as in Figure 3a of *Peltier et al.* [2007].

[26] Following our analyses of the impact of these simple supercontinental geometries, a more realistic paleogeography based on the reconstruction of *Li et al.* [2008] for 720 Ma will also be considered. This geography (Figure 1b) has a total continental area of ~110 × 10^6 km^2, smaller than that employed by *Peltier et al.* [2007] (~130 million km^2, Figure 1a) and modern (~150 million km^2), but very close to that of the smaller of the elliptical models described previously. The reason of this reduction in surface area in *Li et al.*'s [2008] model is either that the continents had yet to be fully formed or that the authors were uncertain where to place several of the less substantial terrains (e.g., West Antarctica) owing to the age uncertainties (Z. X. Li, personal communication, 2007). The continents are more strongly clustered at low latitude in *Li et al.*'s [2008] model compared to that for 570 Ma (Figure 1c). As in the case of the simple elliptical models we will rotate this more realistic supercontinental structure in steps of 10° (Figure 4), to investigate whether or not and in what latitudinal range a hysteresis loop may continue to exist. The sensitivity of the nature of the solutions under such rotational displacements is investigated for several reasons: (1) There are only ~10 samples available between 800 and 700 Ma, for which both the age and paleolatitude were determined, and these are unevenly distributed on the continental fragments [*Trindade and Macouin*, 2007; *Li et al.*, 2008]. (2) The paleolatitude of the individual continents determined by paleomagnetic data is known only to within rather broad error bounds, especially if the sampling is on sedimentary rocks [*Tauxe and Kent*, 2004; *Tauxe et al.*, 2008]. (3) Owing to the scarcity of the data points and the uncertainty in both the age and paleolatitude of the measurements, the positions and/or orientations of particular continental fragments are highly uncertain and therefore controversial (as stated by *Li et al.* [2008], p. 180, "not every opinion expressed in this paper is agreed on by all co-authors"). (4) It will be useful to know over what range of latitudes the center of mass of the supercontinent is located wherein the "oasis branch" of the hysteresis loop may continue to exist.

[27] Aside from the difference in the total continental area and in the latitudinal distribution of the continents between the 720 Ma geography and the Marinoan-like geography, we will demonstrate the existence of a further fundamental difference between the two geographies insofar as the nature of the solutions that they support is concerned. Insofar as the Marinoan-like geography is concerned, the individual continental fragments are more tightly connected. For this reason we will also consider a slightly modified version of the 720 Ma geography (Figure 5), in which the three major continental fragments are modified so as to more tightly connect them. This will involve the addition of ~3 × 10^6 km^2 of additional continental area which is well within the uncertainty of the original reconstruction [see *Li et al.*, 2008, Figure 10] (Z. X. Li, personal communication, 2007). This is a reasonable modification of the paleogeography to contemplate since when large continental ice sheets form in our model, sea level may drop by several hundred meters. The shallow seas that may have separated the individual continents could therefore plausibly have become dry land, thus modifying the surface over which the ice is able to advance without being subjected to calving.

[28] For the more realistic continental configuration, the sensitivity of the existence of oasis solutions upon the precipitation rate will also be further tested by investigating the sensitivity of the space of steady state solutions to variations in the assumed precipitation rate. For the purpose of these additional analyses, precipitation rate will be varied by increments of 0.1 m yr^{-1} up to 0.9 m yr^{-1} from a minimum of 0.2 m yr^{-1}, but these analyses will not be performed for all latitudinal distributions of the supercontinent.

4. Results and Discussion

4.1. Simple Elliptical Supercontinents

[29] The state space of steady state solutions for the ice sheet coupled climate model is calculated for a series of latitudinal positions of the pair of elliptical supercontinents. Of

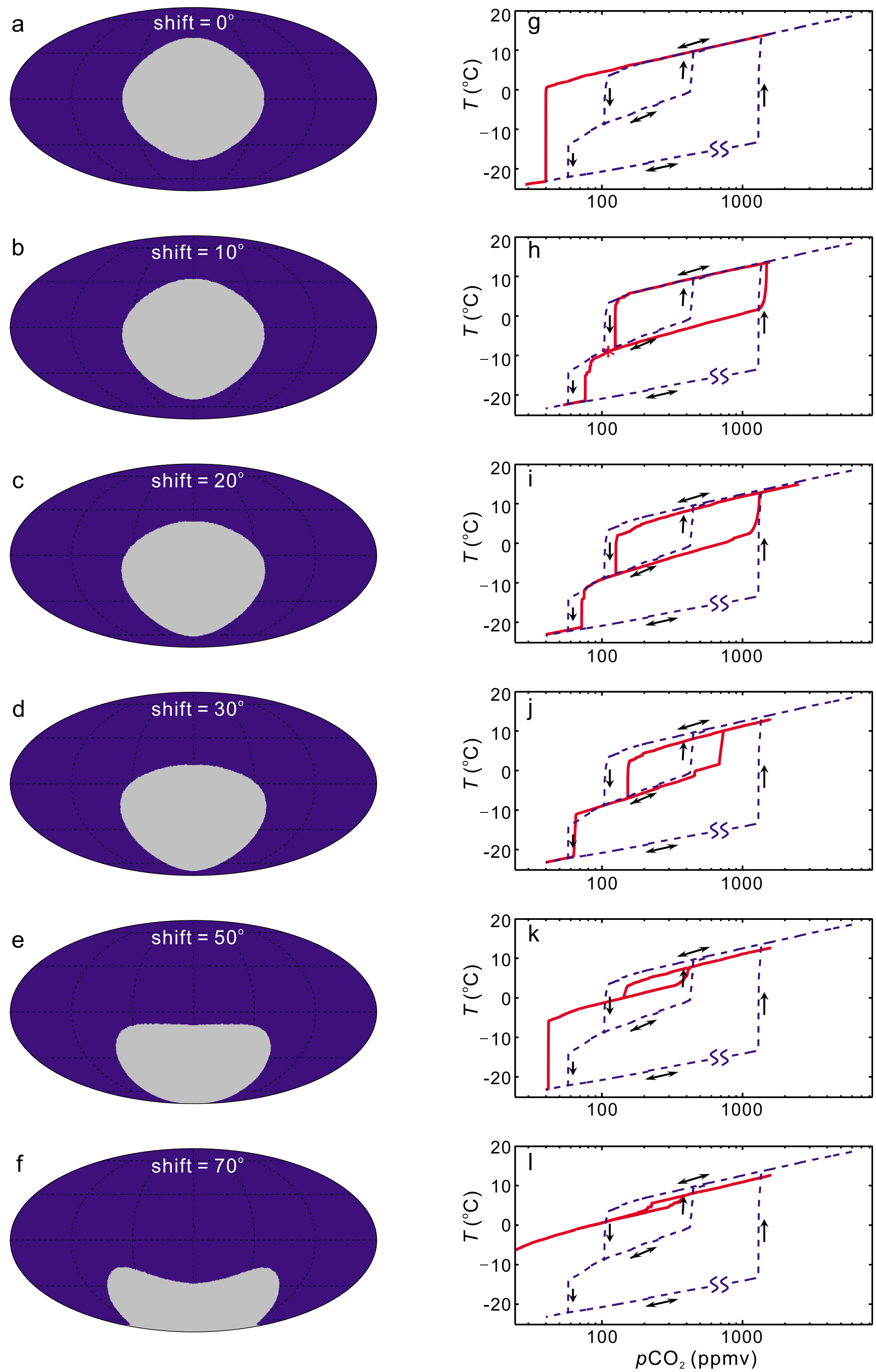

Figure 2. (a–f) The smaller (~105 × 10^6 km^2) of the two elliptical supercontinents centered at a series of different latitudes. (g–l) The hysteresis loop (closed orbit of the red solid lines) of the climate model corresponding to the adjacent paleogeography. The blue dashed lines, upon which the set of steady state solutions for the new set of models are shown in red, are shown for reference from *Peltier et al.* [2007, Figure 3a] for the Marinoan model of Figure 1a.

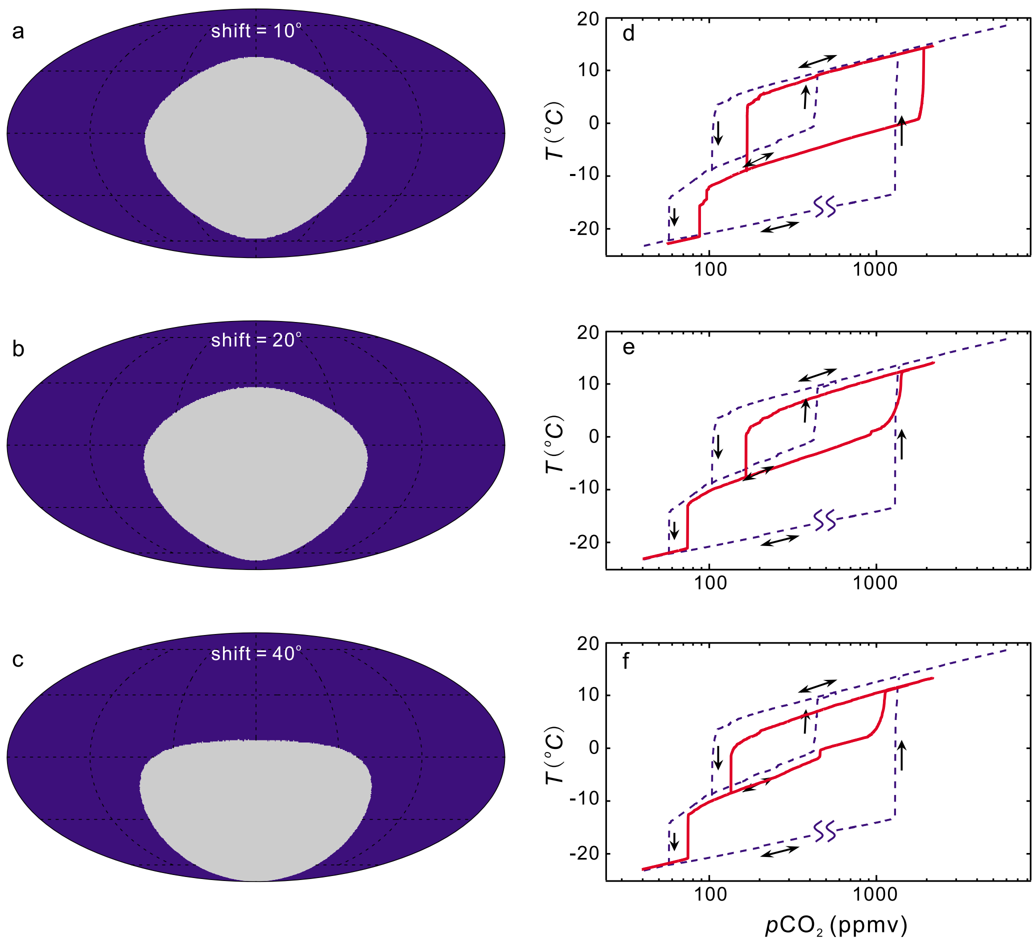

Figure 3. Similar to Figure 2 except that these are for the larger (~130 × 10^6 km^2) of the two elliptical supercontinents.

special interest is the question as to whether proximity of the supercontinent to the pole has any impact upon the existence of the hysteresis loop that is prerequisite for the existence of oscillatory solutions to the fully coupled model that includes an active carbon cycle. The results for the smaller (~105 × 10^6 km^2) elliptical supercontinent centered at 0°, 10°S, 20°S, 30°S, 50°S and 70°S are shown in Figure 2, respectively. In these runs, the solar constant has been taken to be 7.2% lower than the present value, since we are focusing upon the Sturtian glaciation which occurred at approximately 720 Ma. As previously mentioned, it has been well established through analyses of the evolution of main sequence stars that the luminosity of such objects, of which our Sun is an example, increases with time at a rate of ~1% per 100 million years [e.g., *Gough*, 1981].

[30] Figure 2g demonstrates that when this supercontinent is centered precisely on the equator, the oasis/soft snowball state no longer exists as the hysteresis in the system is eliminated. Climate simply descends into a hard snowball state from a relatively warm state (T is around 1°Celcius) when pCO_2 is very low (~40 ppmv). That the climate is stable for very low pCO_2 (but >40 ppmv) is a consequence of the fact that land ice cannot spontaneously develop on any part of the supercontinent owing to its low latitude. There is therefore no positive feedback possible from land ice albedo. When the same model supercontinent is shifted southward by only 10°, however, the nature of the solution space of the model changes significantly. Figure 2h demonstrates that when the supercontinent is centered at 10°S, the climate first enters an oasis state when pCO_2 is gradually decreased, before descending into a hard snowball state, a feature that is the hallmark of the existence of hysteresis. At this position, more than half (68%, or 71 × 10^6 km^2) of the land is still in the equatorial region, that is, between 23°S and 23°N. This, somewhat counterintuitively, suggests that even continental-scale glaciation in the equatorial region does not guarantee the occurrence of a “hard snowball” Earth. It is possible that the oasis solution exists when the supercontinent is located between 0°S and 10°S, but we have not attempted to refine our computation of the critical latitude since the existing results are sufficient to serve the purpose, as will be further discussed in what follows. The reason why oasis solutions are recovered when the continent is shifted southward by 10° will also be further discussed.

[31] As the supercontinent is shifted further and further toward the South Pole, the oasis solution with its characteristic hysteresis loop continues to exist but the area of the loop shrinks, as demonstrated in Figures 2i–2l. When the continent is centered at 50°S, the hysteresis loop (Figure 2k) is already too compressed to be meaningful as short timescale natural internal variability of the climate system, such as that which characterized Pleistocene glacial cycles, could shift the cli-

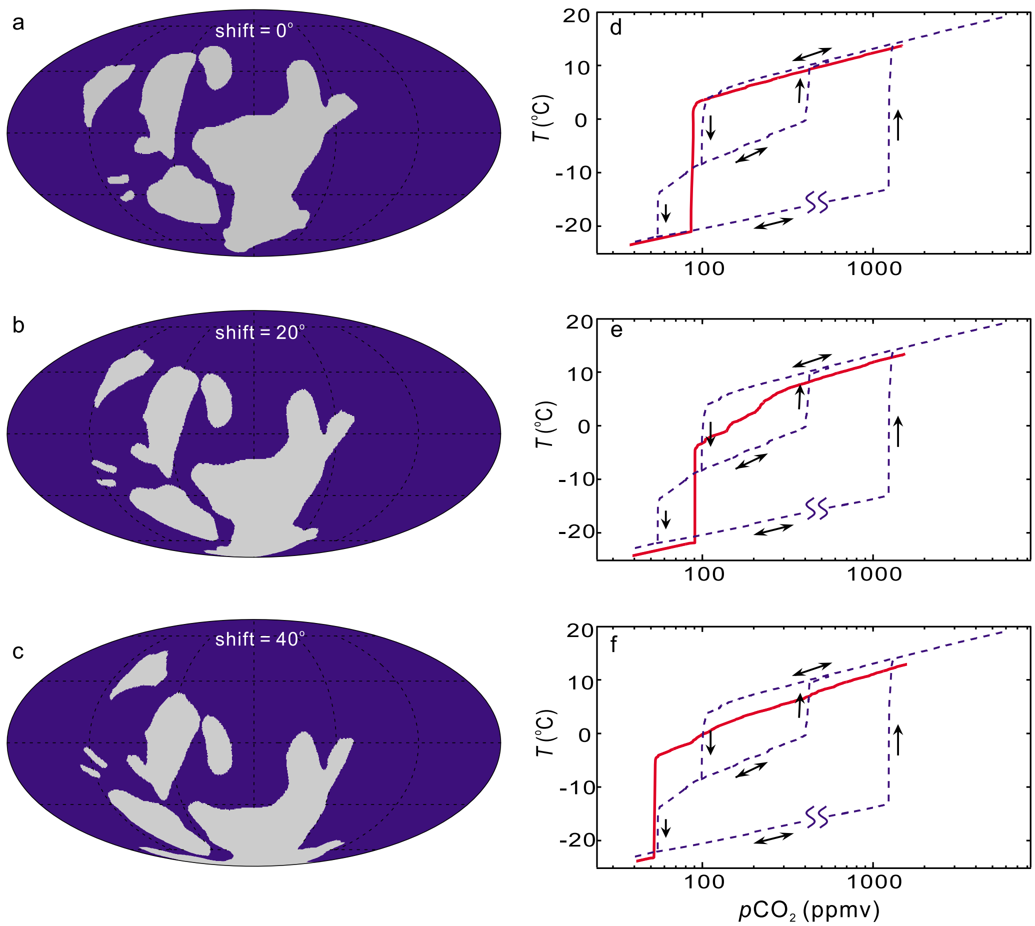

Figure 4. Similar to Figure 2 except that the geography is the more realistic one from *Li et al.* [2008] for 720 Ma.

mate from one branch to the other. Therefore, when the supercontinent is largely located in the high-latitude region, the oasis solutions exist but are no longer distinguished from the hot branch solutions. This shrinking of the hysteresis loop is caused by both a warmer oasis state and a colder hot state, both due to the existence of the continental ice sheet at high latitude. The land ice sheet develops when $p\mathrm{CO}_2$ is high, therefore the hot state becomes slightly colder owing to this albedo effect. However, when the continent is fully glaciated, this albedo effect is smaller when the continent is at high latitude compared to when it is at low latitude, hence a warmer oasis state. It is obvious that oasis solutions most naturally develop if the continent is primarily at high latitude and exist for a wide range of $p\mathrm{CO}_2$. These states are not strongly distinguished from the hot states because large ice sheets have already formed in the hot state as is the case for the Antarctic ice sheet under present-day conditions. Even when the entire continent is glaciated, mean surface temperature is not strongly diminished owing to the reduced impact of ice albedo feedback when the ice-covered region is at high latitude. These high-latitude cases are of no particular interest as too little land lies in the equatorial region to be relevant to the debate concerning Neoproterozoic glaciation theories.

[32] For the larger (~130×10^6 km^2) elliptical supercontinent, the results are very similar. The hysteresis loop also appears when the continent is shifted by 10° toward the South Pole (with 63% or 83×10^6 km^2 of the continent in the equatorial region), but completely disappears when it is shifted 50° or more southward. Only the results for the cases in which the continent is shifted 10°, 20° and 40° are shown explicitly (Figure 3). Overall, it is clear that a distinct oasis state with its attendant hysteresis loop exists for a range of latitudinal distributions for a supercontinent which has a simple geometric form. The continental configuration during the Marinoan glaciation had an area of ~130×10^6 km^2, and was centered around 40°S (Figure 1a), thus a distinct hysteresis loop was found to exist by *Crowley et al.* [2001] and *Peltier et al.* [2004, 2007], consistent with the results for the larger elliptical supercontinent considered here.

[33] It might therefore appear on the basis of the above analyses that an equatorially centered supercontinent may prevent the existence of an oasis solution due to runaway ice albedo feedback entirely, but this turns out not to be the case. As is demonstrated more clearly in section 4.2, if an oasis state is to form, a large ice sheet must first be initiated at high latitude so that it may thereafter expand to low latitude by flow even when the local temperature remains high. When the high-latitude portion of a supercontinent is insufficiently extensive, however, an ice sheet is not easily initiated nor can it grow to sufficient size to enable it to flow toward the equator. This is clearly demonstrated in Figure 6, in which we can see that the volume of land ice is zero if the smaller

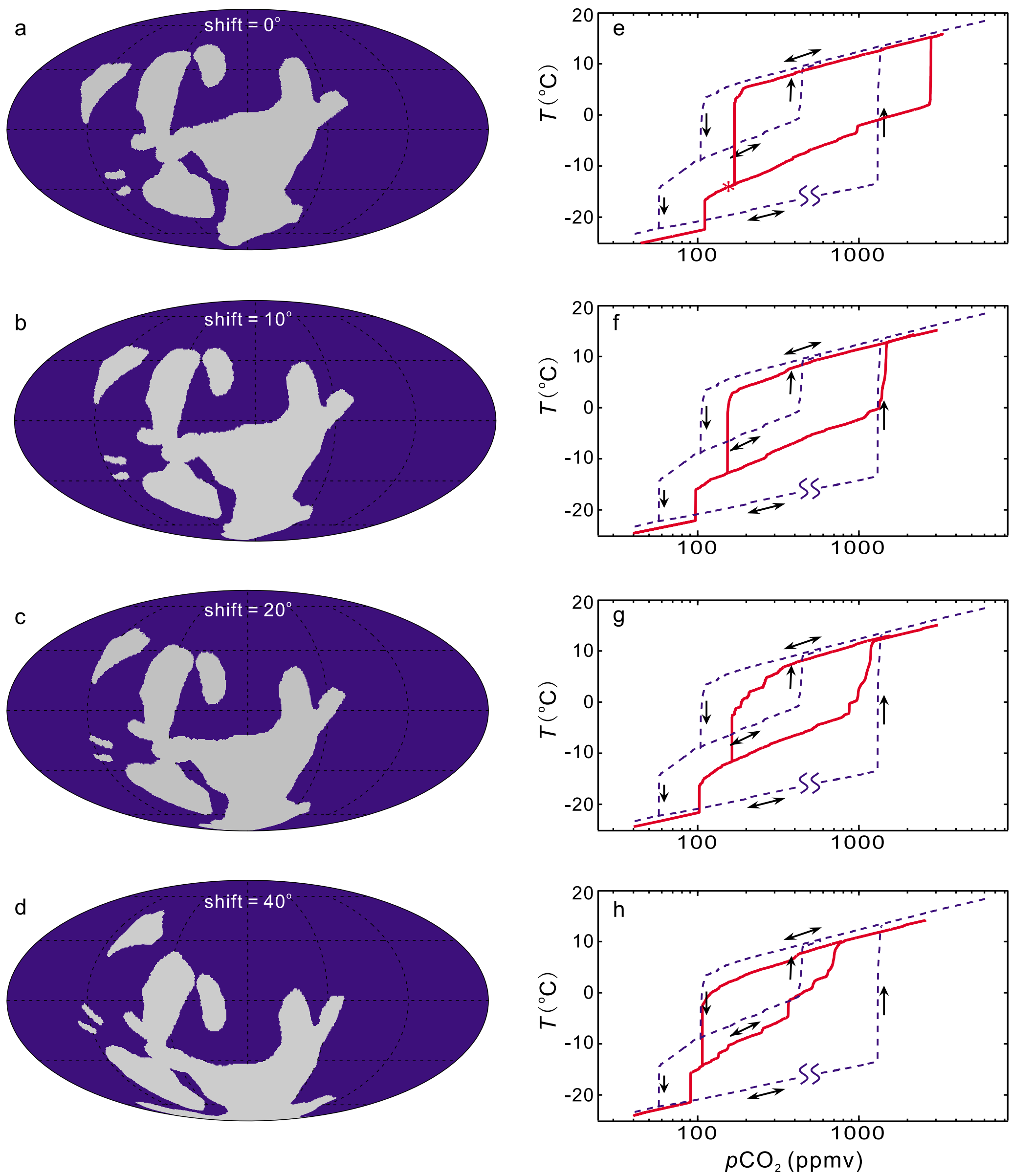

Figure 5. (a–h) Similar to Figure 4 except that the geography is modified slightly to increase the connectivity between the continental fragments.

supercontinent is located at the equator even when pCO_2 is very low (40 ppmv, Figure 6a), but if the continent is shifted southward by 10°, land ice starts to grow at much higher pCO_2 (140 ppmv, Figure 6b), and it is even more obvious when the continent is shifted southward by 20°. This is true for both of the two elliptical supercontinents. In both cases the oasis solution is recovered when the supercontinents are shifted from 0° to 10°S, during which the equatorial portion of the continent is only decreased from 69% (Figure 2a) to 68% (Figure 2c) and from 63.2% to 62.5% (Figure 3a) of the total area, respectively. This would not be expected to cause any significant change insofar as the ice albedo effect is concerned. However, by extending the southern edge of the continent by 10° south, the high-latitude land area is just sufficiently large enough to enable low-latitude glaciation to develop. An ice sheet appears at pCO_2 of ~140 ppmv, slightly higher than the critical pCO_2 (~110 ppmv, Figure 6b), and flows very slowly owing to its small size. Because the equilibrium mass of the ice sheet for a slightly lower pCO_2 is much larger, the model requires increased time to reach steady state when pCO_2 is decreased. If pCO_2 is deceased further when the ice sheet is still far from equilibrium, the pCO_2 will be excessively low to allow the oasis solution to develop and a "hard snowball" is obtained. In our initial application of the approximate technique for defining the hysteresis loop which employed the excessively rapid rate of pCO_2 reduction of 1 W m^{-2}/50 kyr, this led to our missing the existence of the oasis solution. The rate that was adequately slow for analysis of the Marinoan continental distribution was inadequate in the present circumstance.

[34] The variation of the critical pCO_2 value (Figures 2g–2l), at which transition to a "hard snowball" occurs as a function of the latitudinal distribution of the supercontinent is also related to the development of the continental ice sheet.

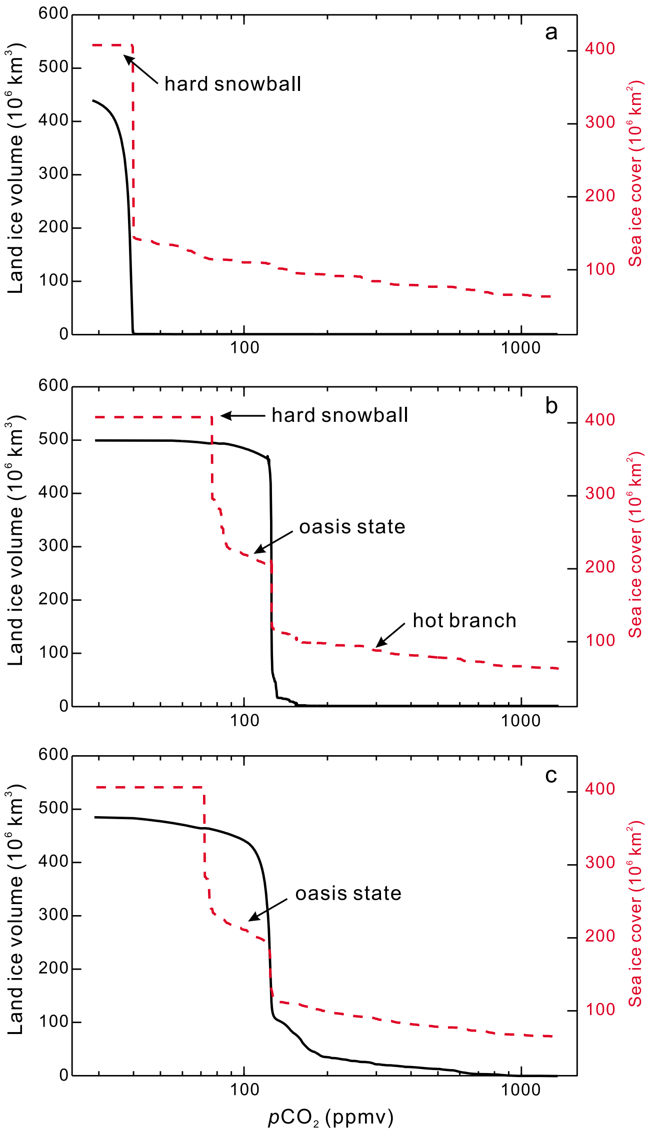

Figure 6. Total land ice volume (black solid line) and sea ice area (red dashed line) of a series of steady states when pCO_2 is continuously decreased, for the smaller elliptical supercontinent centered at (a) the equator, (b) 10°S, and (c) 20°S.

Development of a land ice sheet at high latitude at relatively high pCO_2 assists the development of a "hard snowball," resulting in a higher critical pCO_2 value (compare Figures 2h and 2i with Figure 2g). However, when the supercontinent is at very high latitude, the critical pCO_2 is lower because the albedo effect of the land ice is reduced (compare Figure 2j with Figure 2k).

4.2. More Realistic 720 Ma Sturtian Paleogeography

[35] Similar to the procedure employed above, the more realistic continental configuration appropriate for 720 Ma (Figure 1b) is applied for this model in the search for steady state solutions. It is found (using the more accurate method of defining the state space of steady state solutions) that for all latitudinal distributions of the *Li et al.* [2008] arrangement of the continental fragments, the climate descends directly into the hard snowball state when pCO_2 is reduced to a sufficiently low level (Figure 4). This could be taken to contradict the results described in section 4.1 since the total area of the continents in this case is midway between that for the two elliptical supercontinents and closest to the smaller. However, we note that the 720 Ma continental configuration differs from that of the elliptical supercontinent in at least two respects. The shape of the overall envelope of individual continental fragments as well as the much reduced degree of connectivity is strikingly different between these cases. While there is no reason to believe that the shape of the outline of the supercontinental envelope should be especially determinant of the nature of the solutions, the connectivity between individual continental fragments could be critical as, to the extent that the individual continental masses are more isolated, the flow of ice over the supercontinent will be strongly inhibited.

[36] When the Sturtian continental configuration is slightly modified as in Figure 5 so that the connectivity is increased, it is found that a distinct hysteresis loop reappears even when the supercontinent is not shifted in latitude (Figure 5e). The hysteresis loop again disappears, however, when the fragments of the supercontinent are collectively shifted poleward by more than 60°. The most significant difference between the hysteresis loops obtained for different latitudinal shifts of the supercontinent may be that the pCO_2 level required for escape (~1200 ppmv) from the "soft snowball" state is higher if the supercontinent is centered on the equator (Figures 5a and 5e), and occurs much more abruptly than if the supercontinent is at higher latitude (e.g., Figures 5g and 5h). There is greater complexity of the set of steady states that define the hysteresis loop when the supercontinent is at higher latitude (Figures 5g and 5h), which makes the transition between the hot state and the oasis state much less abrupt, and therefore less problematic to define when the previous imprecise method is employed to define the set of steady states.

[37] When the previous imprecise approach in which the rate of pCO_2 variation was overly rapid was employed, the oasis solutions were missed for the two cases in which the continents are either at the equator or shifted southward by 10° (Figure 7). It is especially clear by comparing the red lines in Figures 7a and 5e that pCO_2 was decreased too rapidly in the run of the time-dependent model in the construction of Figure 7a, such that when the land ice sheet reaches equilibrium pCO_2 is already close to the critical value (Figure 5e) at which a "hard snowball" transition would occur.

[38] Although the hysteresis loop exists for all the latitudinal distributions of the supercontinent that are of interest here, the appearance of such multiple equilibria is also sensitive to the magnitude of the precipitation rate assumed (Figure 8). The data presented on Figure 8 demonstrate that if oasis solutions exist for a specific precipitation rate, then they exist for a higher precipitation rate. This is clearly related to the fact that at higher precipitation rate, ice sheet mass accumulates more quickly if an ice sheet forms at all and is therefore better able to sustain flow over the landscape. The opposite is also true; if oasis solutions do not exist at some specific precipitation rate, they will not be found at any lower precipitation rate. It is also seen that when the realistic supercontinent remains at its equatorially centered location, oasis solutions cannot be found when the precipitation rate is lowered by 0.1 m yr^{-1} to 0.5 m yr^{-1}, but if the supercontinent is shifted toward the South Pole by 5°, the oasis solutions

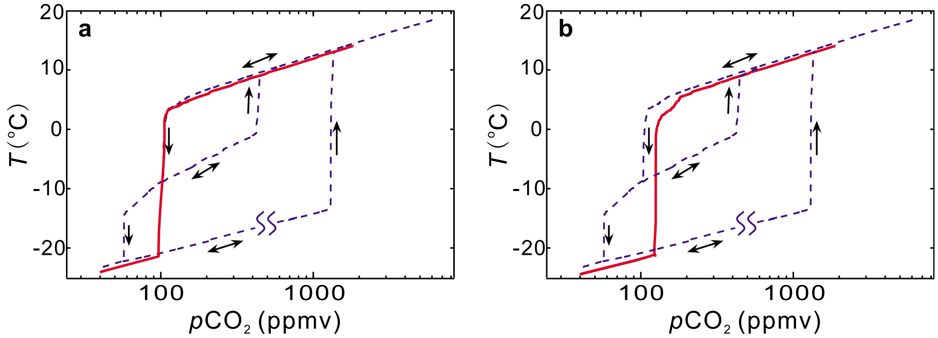

Figure 7. The same as (a) Figure 5e and (b) Figure 5f, respectively, except that the hysteresis loops here are calculated with the imprecise method previously used by *Peltier et al.* [2004] in their analysis of the Marinoan supercontinent, in which the pCO_2 was varied too quickly to enable accurate characterization of the hysteresis loop.

are found for the precipitation rate of 0.5 m yr^{-1}, but not 0.4 m yr^{-1}. Only when the supercontinent is shifted southward by 15°, will the oasis solutions exist for a precipitation rate of 0.4 m yr^{-1}. The reason that the oasis states are able to develop at lower precipitation rate when the supercontinent is at higher latitude is also related to the nucleation and growth rate of the ice sheet when it initiates at the high latitude. Further refinement of the variations of the precipitation rate are unwarranted given the simplicity of the climate model that we are employing. In the future we intend to further investigate whether "soft snowball" steady states exist by imposing a precipitation field obtained from a full general circulation model (GCM) run at appropriate boundary conditions as in the work of *Peltier et al.* [2004].

[39] Land ice and sea ice distributions for two examples of oasis solutions, for the smaller elliptical supercontinent and the realistic 720 Ma continental configuration, respectively, are shown in Figure 9. From Figure 9, we can see that a large area of open ocean continues to exist even with the low-latitude continents completely covered by ice. These are typical images of a soft snowball. It is also apparent that a significant area of sea ice is formed around the glaciated continental coasts and in the equatorial ocean enclosed by the continental fragments. An important phenomenon concerning the oasis/soft snowball solutions (in both Figure 9 here and Figure 5 of *Peltier et al.* [2007]) is that the primary sea ice boundaries (not those close to the continents) can be far from entering the Hadley regime. However, owing to the simplicity of the climate model we are employing, it is not possible to

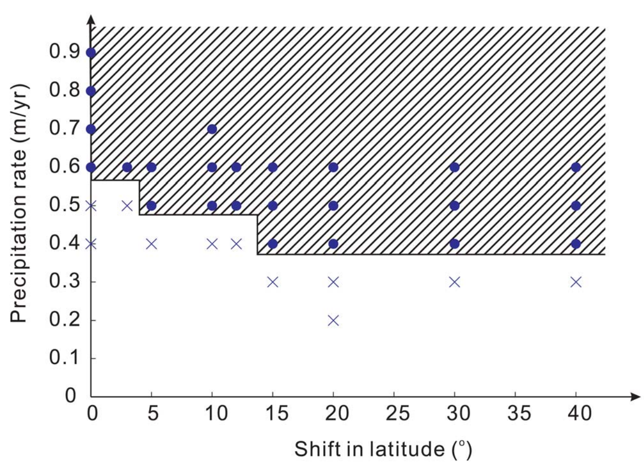

Figure 8. The existence of the "soft snowball"/oasis solutions in the precipitation rate versus latitudinal shift space. In the hatched area, oasis solutions exist. The specific tests performed to delineate the boundary between these two types of solutions are shown as solid circles (for which oasis states exist) and crosses (for which oasis states do not exist).

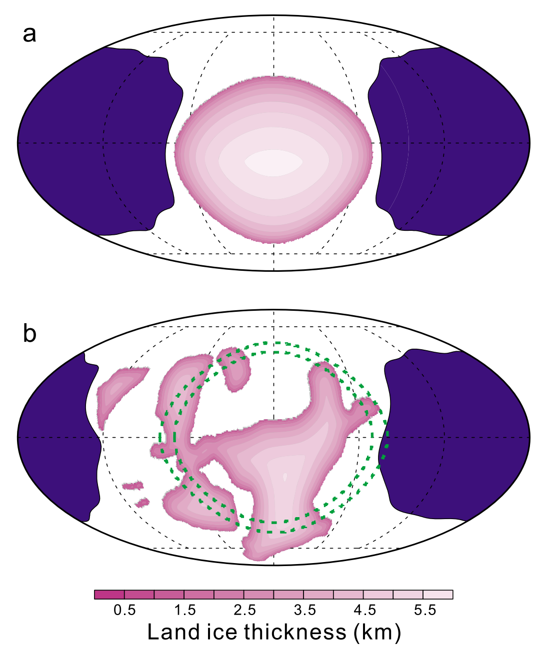

Figure 9. (a) Ice distribution in oasis solutions for the smaller elliptical supercontinent shifted by 10° southward (Figure 2b). (b) For the more realistic 720 Ma supercontinent (Figure 5a). Pink area represents the land ice, white area represents the sea ice, and blue area represents the ocean. The locations of the solution in the mean temperature–pCO_2 state space are indicated by asterisks in Figures 2h and 5e, respectively. The dashed green lines in Figure 9b are the outlines of the smaller (inner) and bigger (outer) unshifted elliptical supercontinents, respectively.

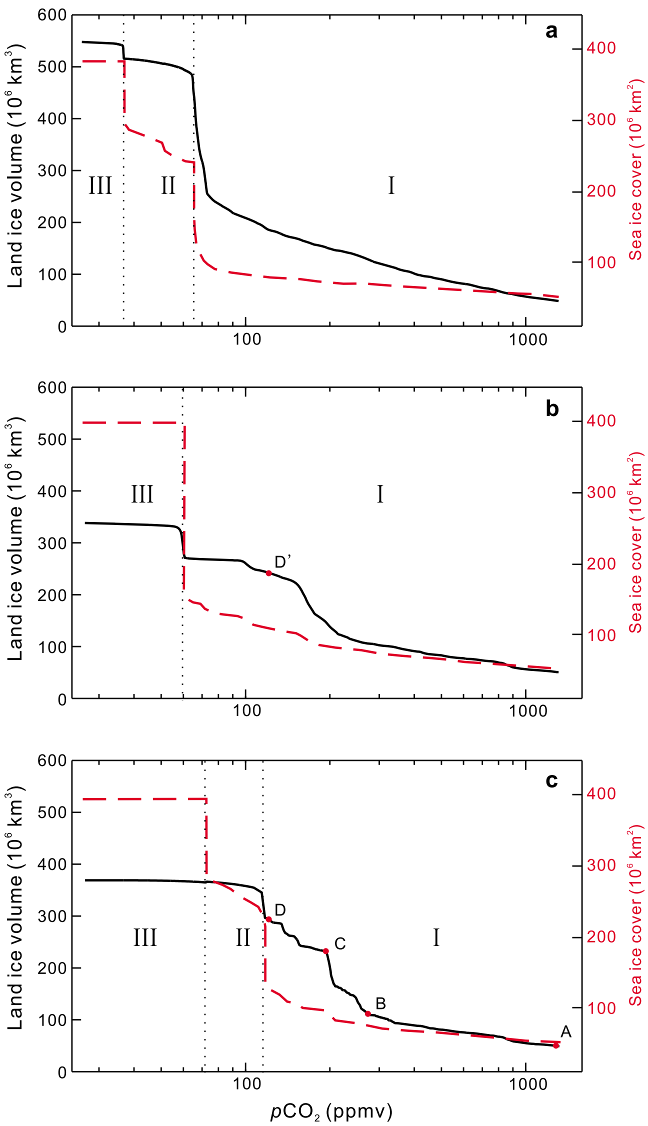

Figure 10. Total land ice volume (black solid line) and sea ice area (red dashed line) of a series of steady states when pCO_2 is continuously decreased for (a) 570 Ma geography, (b) 720 Ma geography, and (c) 720 Ma geography with both increased connectivity and a 20° southward shift, respectively.

assert that in a "soft snowball" state, sea ice may never extend to within 30° of the equator.

[40] The difference in latitudinal distribution between the simple elliptical supercontinents and the more realistic one is also illustrated in Figure 9. The southern most edges of the smaller and bigger elliptical supercontinents (see the dashed green lines in Figure 9b) reach ~42°S and ~48°S, respectively, while that of the more realistic supercontinent reaches ~75°S, significantly more fully extended into the polar region. Since the hysteresis loop can be recovered for both of the elliptical supercontinents by only a 10° shift, therefore, it is not surprising that the hysteresis loop exists for the more realistic supercontinent, on which the land ice sheet can form at relatively high pCO_2. The land area of the more realistic supercontinents south of 42°S and 48°S are 17×10^6 km^2 and 11×10^6 km^2, respectively, if fully glaciated, the size of the ice sheet will be comparable or larger than the Laurentide ice sheet during the last glacial maximum, and capable of fast flow.

[41] Our results suggest that oasis solutions exist even when a large fraction of the total continental area (at least 60% for the elliptical supercontinent and 50% for the more realistic 720 Ma continents) lies in the equatorial region. We have also demonstrated that the latitudinal distribution of continents is important in order for the oasis solutions to exist at lower precipitation rate, but the most critical prerequisite for the existence of a "soft snowball" is that sufficient connectivity exists between the individual continental fragments so that land ice may flow between them so as to increase the total area covered. This is clearly seen by comparing Figures 4 and 5, but the detailed demonstration of this fact is provided in the following discussion of Figures 10 and 11.

[42] Figure 10 shows the total land ice volume and sea ice area for a range of pCO_2 for three different geographies, namely the Marinoan-like 570 Ma geography (Figure 1a), the 720 Ma geography shifted by 20° southward (Figure 4b) and the 720 Ma geography shifted southward by 20° and with increased connectivity of the continental fragments (Figure 5c), respectively. The reason why we do not choose to compare the simulations of Figures 4a and 5a is that there is a much reduced range of steady states existing between the hot state and oasis state in these two cases, and land ice does not start to grow until pCO_2 is low and then quickly transforms into a "soft snowball." The comparison is therefore unsuitable for demonstrating the importance of ice flow as clearly as for this set of 3 paleogeographies. The individual curves shown in Figure 10 for each of these three paleogeographies are divided into three regimes: the partial glaciation regime (I), the soft snowball regime (II), and the hard snowball regime (III), respectively. The first difference that is evident by comparing these results for the individual paleogeographies is that the maximum total land ice volume (in regime III) in Figure 10a is much larger than in the other two cases. This is because the larger total continental area and very good connectivity of the 570 Ma continental configuration, which tends to have a more spatially extensive and thicker ice sheet. The second difference is that the land ice curve in regime I of Figure 10a is much smoother than those in Figures 10b and 10c. This is because the much increased complexity of land distribution in Figures 4b and 5c relative to that of Figure 1a. This also causes the hysteresis loop (red line) in Figure 5g to be more complex than that for the 570 Ma geography (blue dashed line in Figure 5g). The most important difference is between Figures 10b and 10c. The two models are only different in continental connectivity, but in Figure 10c, the land ice volume reaches its maximum at a much higher pCO_2 level than that in Figure 10b. At both point D′ of Figure 10b and point D of Figure 10c, the land ice volumes are similar, but in Figure 10c, the ice volume strongly transitions into a soft snowball regime when pCO_2 is decreased only slightly. Figure 11d clearly demonstrates that at this point, land ice has reached a critical state and the ice sheet is just about to expand onto the low-latitude continent. However, for the model in Figure 10b, land ice is restrained from flowing onto the low-latitude continent and so it requires a much lower pCO_2 so that ice can spontaneously expand locally, which implies that sea ice will also be able to expand spontaneously in the

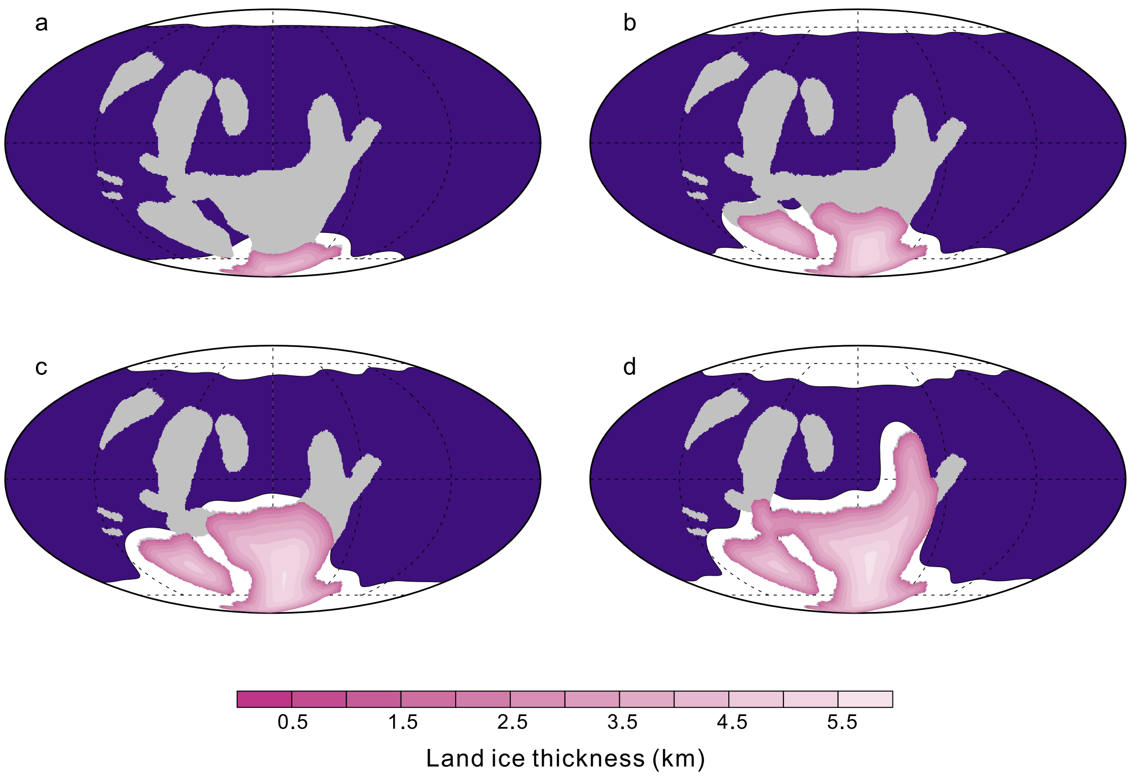

Figure 11. Ice distribution in the four steady states indicated by letters “A” to “D” in Figure 10c.

low-latitude ocean. This is what occurs in the transition between regimes I and III in Figure 10b, so no regime II exists for this model.

[43] The importance of ice flow is not just critical at the transition point between regimes I and II in Figure 10, but it is also important in the entirety of regime I. At the beginning of regime I, the ice sheet is small (Figure 11a) so that the flow of ice is not significant and the expansion of the land ice sheet and sea ice seem to progress at a similar pace as $p\mathrm{CO}_2$ is decreased. When the scale of the ice sheet reaches a critical value (Figure 11b), the expansion of the land ice sheet due to ice flow becomes significant, so that with only a slight decrease of $p\mathrm{CO}_2$, the ice sheet is able to expand significantly, as demonstrated in Figure 10c between points B and C. When the ice flow is restricted by the land configuration, such as in Figure 11c, ice growth becomes much slower as $p\mathrm{CO}_2$ is decreased, as evident in the slope of the solid curve between points C and D in Figure 10c. The reason for this is that on the narrow neck of land, ice flow from high latitude is not high enough to overcome the ablation. Overall, ice flow is critically important for generating land ice in the low-latitude region, which requires that the continental fragments are connected. We have shown in Figure 5 that even a narrow connection between the continental fragments could make a fundamental difference.

[44] At the transition point between regimes I and II in Figure 10, sea ice area is also shifted to a much higher value. This may be attributed to the cooling of the ocean surrounding the glaciated continents due to the diffusive heat transport that is assumed in the EBM to represent the influence of atmosphere dynamical processes. Farther from the continents, the ocean remains warm because $p\mathrm{CO}_2$ is still high, and sea ice cannot form spontaneously within ~40 degree of the equator. Only when the $p\mathrm{CO}_2$ is further decreased below some critical value, will sea ice form in the tropical region and eventually close at the equator owing to runaway sea ice albedo feedback (transition from regime II to III in Figure 10). This also explains why the low-latitude fraction of continental area cannot exceed a maximum value if oasis solutions are to exist. When too much land lies in the equatorial region, it has three consequences: (1) The equatorial ocean area is itself reduced. (2) When the land becomes glaciated by ice flow, there is an enhanced impact upon climate due to the change in surface albedo. (3) Enhanced sea ice formation occurs in the surrounding oceans owing to diffusive heat transport in the model atmosphere. Therefore, the already reduced equatorial area of the ocean may more easily be covered by sea ice, thereby resulting in a hard snowball. The shape of the continental coastline is obviously also very important. For example, for the 720 Ma supercontinent, much more sea ice is formed on the ocean surface (Figure 9b) than if the supercontinental configuration has less small-scale variation and the continental fragments are more compact (as in Figure 9a).

[45] As demonstrated above, to simulate the development of an oasis state, an active ice sheet module must be included in the model. However, because the ice sheet equilibrates only slowly with climate, it remains premature at the present to attempt to test the plausibility of these EBM-coupled ice sheet model simulations using a General Circulation Model (GCM). This fact is reinforced by Figure 12, which shows that if a model is initialized from a zero-ice state, following which $p\mathrm{CO}_2$ is suddenly decreased to a low value ($d$rad = -4 W m^{-2}), it takes more than 100 kyrs for the ice sheets to cover the supercontinent and the climate to enter an oasis

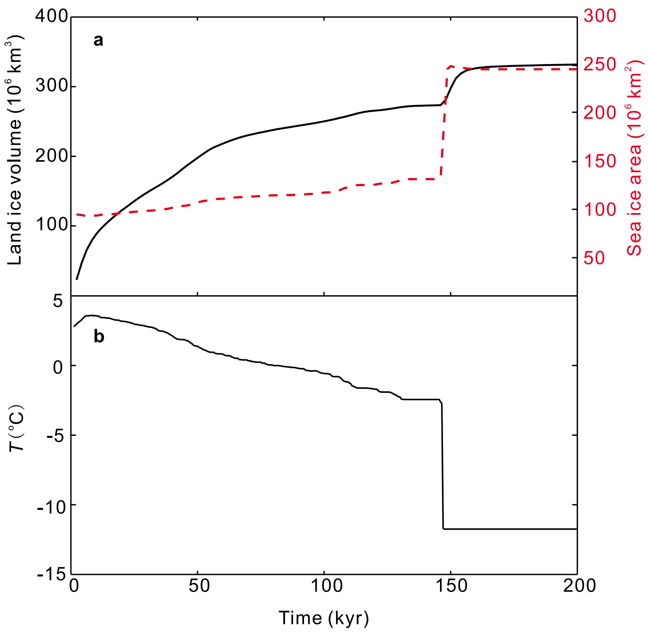

Figure 12. (a) Time series of land ice volume and sea ice area for a run in which the model starts with a zero-ice state but $d$rad is suddenly decreased and fixed to −4 W m^{-2}, until the steady state (an oasis state) is reached. The geography in Figure 5c is employed. (b) Time series of temperature for the same run.

state. The geography used in this simulation is the same as Figure 5c. It will also be seen that the major temperature drop (at ~150 kyr) is associated with the sudden expansion of sea ice (around the continents at the equator) because of the strong positive feedback, while the final major expansion of land ice at ~150 kyr acts as a trigger by first generating sea ice in the ocean surrounding the glaciated continents at the equator.

[46] Although we have argued that a possible drop of sea level due to the growth of land ice may help connect the continental fragments, this process has not been explicitly modeled. Rather, what we have done is to add additional continental freeboard to connect them. Consideration of the influence of a fall in sea level is significant in helping us understand the process of "soft snowball" formation. One might well ask what the effect might be if the impact of the fall in sea level were computed directly on the basis of the land ice volume. However, detailed information regarding the continental shelves would be required to fully investigate this question and in reality even the continental boundaries themselves cannot be precisely determined [*Li et al.*, 2008]. When we input the paleogeography reconstructed by *Li et al.* [2008], we have already enhanced the continentality slightly so that our continental boundaries enclose theirs. It would not be possible at the present time to significantly improve our understanding of the processes involved in "soft snowball" formation by explicitly including in the model a description of the link between continental ice sheet growth and the increase in continental freeboard for this reason.

[47] Since the condition for the formation of a soft snowball is that the grounded continental ice sheet is able to expand from high latitude to low latitude by ice flow, the connectivity between the continental fragments may be weaker than that shown in Figure 5. Actually, calculation shows that a slight modification of the 720 Ma configuration, as shown in Figure 13, is sufficient for the oasis solution to be recovered. With this modification, the fragment of the supercontinent at low latitude is connected to only one of the two high-latitude fragments as opposed to the modification shown in Figure 5. The connectivity requirement is thus much weaker in the sense that an entirely compact supercontinent may not be required for the existence of steady state oasis (soft snowball) solutions.

[48] Another way to increase the connectivity between the individual continental fragments is simply to shift several of the smaller fragments along the latitude circle at which they are initially positioned so that they are close to the larger fragments. When sea level falls owing to the development of high-latitude ice sheets, they may then be effectively connected to each other. This is not constrained by the paleomagnetic measurements, since these constrain only the paleolatitude of the terrain in which the measurements were made. The relative position of the continents on the same latitude circle must be determined using other geologic considerations (e.g., sedimentological data). A modified geography constructed in this way was also tested, and the results

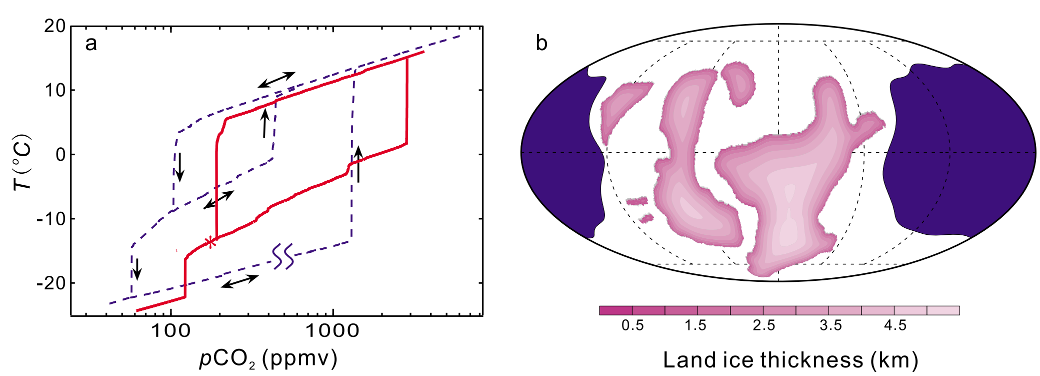

Figure 13. (a) The hysteresis for the model in which the continental fragments are connected in a different way from that in Figure 5. Only two of the three major fragments are connected. (b) The ice distribution in an oasis state for this geography as indicated by an asterisk in Figure 13a.

obtained (not shown) were similar to those shown in Figure 5, the hysteresis loop of steady state solutions is once again recovered.

[49] The continental configuration (for 750 Ma) suggested by *Trindade and Macouin* [2007] for the late Neoproterozoic is not drastically different from the 720 Ma paleogeography employed herein. We therefore expect that with appropriate minor adjustment of the connectivity of the continental fragments within the error bars shown in their paper, and shifting in longitude (as allowed by paleomagnetic constraints) of some of the continental fragments, the oasis branch of steady state solutions will once more exist.

5. Conclusions

[50] Through detailed analysis of the formation of a "soft snowball" for various continental configurations, we have demonstrated the importance of land ice flow in the generation of low-latitude land ice. Because the flow of land ice only becomes significant when the size of the land ice sheet is large enough, it requires that the continental area in high latitudes should also be large enough that a massive ice sheet may develop even when pCO$_2$ is relatively high. In order for the land ice sheet initially located at high latitude to be able to flow onto the low-latitude land surface, this obviously requires that the continental fragments in low latitude must be sufficiently connected (or separated only by continental shelves above which water depths are small) with those at higher latitudes. Finally, since sea ice forms in the ocean immediately surrounding the glaciated continents, a relatively simple supercontinental outline is more favorable for the formation of the "soft snowball" state because if too much sea ice is formed around the continents, crossing a threshold value (not quantitatively known), runaway albedo effect will cause the formation of a "hard snowball." Nevertheless, we have demonstrated that the realistic Sturtian continental configuration of *Li et al.* [2008] that existed at ~720 Ma satisfies all of the three conditions above if the connectivity between continental fragments is only slightly modified, and therefore "soft snowball" solutions are found to exist. In our analysis, several of the continental fragments are taken to be separated by shallow oceans and become connected to each other after a sea level drop of hundreds of meters due to land ice growth at high latitude.

[51] Sea ice in the oasis solutions produced by our model (both here and from *Peltier et al.* [2007]) does not extend to latitudes <40° (Figure 9) except in the regions immediately surrounding the glaciated continents, a result that is similar to that reported by *Hyde et al.* [2000] where the issue of the existence of this class of solutions was first raised. However, we cannot infer that this must be true for a "soft snowball" owing to the simplicity of the climate model and low resolution employed here. We have also demonstrated explicitly in the present paper that the flow of land ice is very important to the existence of such solutions, and that the characteristic timescale for this process to equilibrate with climate is ~100 kyr, making the test of the validity of such steady state solutions with GCMs, which include an active ice sheet module, very difficult. Modern coupled climate models which do not include an active ice sheet model cannot be usefully employed to test the possibility that the system is able to enter such a steady state although they may be employed to test its stability as in the work of *Peltier et al.* [2004].

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References

Abbot, D. S., and R. T. Pierrehumbert (2010), Mudball: Surface dust and Snowball Earth deglaciation, *J. Geophys. Res.*, *115*, D03104, doi:10.1029/2009JD012007.

Agassiz, L. (1840), *Etudes sur les Glaciers (in French)*, Jent and Gassmann, Soleure, Switzerland. (English translation, *Studies on Glaciers*, edited by A. V. Carozzi, 244 pp., Hafner, N. Y., 1967.)

Allen, P. A. (2006), Snowball Earth on trial, *Eos Trans. AGU*, *87*(45), 495, doi:10.1029/2006EO450005.

Allen, P. A., and J. L. Etienne (2008), Sedimentary challenge to Snowball Earth, *Nat. Geosci.*, *1*, 817–825, doi:10.1038/ngeo355.

Benn, D. I., C. R. Warren, and R. H. Mottram (2007), Calving processes and the dynamics of calving glaciers, *Earth Sci. Rev.*, *82*, 143–179, doi:10.1016/j.earscirev.2007.02.002.

Butler, S. L., W. R. Peltier, and S. O. Costin (2005), Numerical models of the Earth's thermal history: Effects of inner core solidification and core potassium, *Phys. Earth Planet. Inter.*, *152*, 22–42, doi:10.1016/j.pepi.2005.05.005.

Chandler, M. A., and L. E. Sohl (2000), Climate forcings and the initiation of low-latitude ice sheets during the Neoproterozoic Varanger glacial interval, *J. Geophys. Res.*, *105*(D16), 20,737–20,756, doi:10.1029/2000JD900221.

Christie-Blick, N., L. E. Sohl, and M. J. Kennedy (1999), Considering a Neoproterozoic Snowball Earth, *Science*, *284*, 1087, doi:10.1126/science.284.5417.1087a.

Corsetti, F. A. (2009), Extinction before the snowball, *Nat. Geosci.*, *2*, 386–387, doi:10.1038/ngeo533.

Corsetti, F. A., S. M. Awramik, and D. A. Pierce (2003), Complex microbiota from snowball Earth times: Microfossils from the Neoproterozoic Kingston Peak Formation, Death Valley, USA, *Proc. Natl. Acad. Sci. U. S. A.*, *100*(8), 4399–4404, doi:10.1073/pnas.0730560100.

Corsetti, F. A., A. N. Olcott, and C. Bakermans (2006), The biotic response to Neoproterozoic snowball Earth, *Palaeogeogr. Palaeoclimatol. Palaeoecol.*, *232*, 114–130.

Crowley, T. J., W. T. Hyde, and W. R. Peltier (2001), CO_2 levels required for deglaciation of a "near-snowball" Earth, *Geophys. Res. Lett.*, *28*(2), 283–286, doi:10.1029/2000GL011836.

Dalziel, I. W. D. (1997), Neoproterozoic-Paleozoic geography and tectonics: Review, hypothesis, environmental speculation, *Geol. Soc. Am. Bull.*, *109*, 16–42, doi:10.1130/0016-7606(1997)109<0016:ONPGAT>2.3.CO;2.

Deblonde, G., and W. R. Peltier (1991), Simulations of continental ice-sheet growth over the last glacial-interglacial cycle: Experiments with a one-level seasonal energy-balance model including realistic geography, *J. Geophys. Res.*, *96*(D5), 9189–9215, doi:10.1029/90JD02606.

Deblonde, G., and W. R. Peltier (1993), Late Pleistocene ice age scenarios based on observational evidence, *J. Clim.*, *6*(4), 709–727, doi:10.1175/1520-0442(1993)006<0709:LPIASB>2.0.CO;2.

Deblonde, G., W. R. Peltier, and W. T. Hyde (1992), Simulations of continental ice sheet growth over the last glacial-interglacial cycle: Experiments with a one level seasonal energy balance model including seasonal ice albedo feedback, *Global Planet. Change*, *6*, 37–55, doi:10.1016/0921-8181(92)90021-2.

Donnadieu, Y., G. Ramstein, F. Fluteau, J. Besse, and J. Meert (2002), Is high obliquity a plausible cause for Neoproterozoic glaciations?, *Geophys. Res. Lett.*, *29*(23), 2127, doi:10.1029/2002GL015902.

Donnadieu, Y., Y. Goddéris, G. Ramstein, A. Nédélec, and J. Meert (2004), A "snowball Earth" triggered by continental breakup through changes in runoff, *Nature*, *428*, 303–306, doi:10.1038/nature02408.

Etienne, J. L., P. A. Allen, R. Rieu, and E. Le Guerroue (2007), Neoproterozoic glaciated basins: A critical review of the snowball Earth hypothesis by comparison with Phanerozoic glaciations, in *Glacial Sedimentary Processes and Products*, edited by M. J. Hambrey et al., *Spec. Publ. Int. Assoc. Sedimentol.*, *39*, 343–399.

Evans, D. A. D. (2006), Proterozoic low orbital obliquity and axial-dipolar geomagnetic field from evaporite palaeolatitudes, *Nature*, *444*, 51–55, doi:10.1038/nature05203.

García, H. E., and L. I. Gordon (1992), Oxygen solubility in sea water: Better fitting equations, *Limnol. Oceanogr.*, *37*, 1307–1312, doi:10.4319/lo.1992.37.6.1307.

Goddéris, Y., and Y. Donnadieu (2008), Carbon cycle and snowball Earth, *Nature*, *456*(E8), doi:10.1038/nature07653.

Gough, D. O. (1981), Solar interior structure and luminosity variations, *Sol. Phys.*, *74*, 21–34, doi:10.1007/BF00151270.

Harland, W. B., and D. E. T. Bidgood (1959), Palaeomagnetism in some Norwegian sparagmites and the Late Pre-Cambrian ice age, *Nature*, *184*, 1860–1862, doi:10.1038/1841860b0.

Hoffman, P. F., and Z. X. Li (2009), A palaeogeographic context for Neoproterozoic glaciation, *Palaeogeogr. Palaeoclimatol. Palaeoecol.*, *277*, 158–172, doi:10.1016/j.palaeo.2009.03.013.

Hoffman, P. F., and D. P. Schrag (2000), Snowball Earth, *Sci. Am.*, *282*, 68–75, doi:10.1038/scientificamerican0100-68.

Hoffman, P. F., and D. P. Schrag (2002), The snowball Earth hypothesis: Testing the limits of global change, *Terra Nova*, *14*, 129–155, doi:10.1046/j.1365-3121.2002.00408.x.

Hoffman, P. F., A. J. Kaufman, G. P. Halverson, and D. P. Schrag (1998), A Neoproterozoic Snowball Earth, *Science*, *281*, 1342–1346, doi:10.1126/science.281.5381.1342.

Hoffman, P. F., J. W. Crowley, D. T. Johnston, D. S. Jones, and D. P. Schrag (2008), Snowball prevention questioned, *Nature*, *456*(E7), doi:10.1038/nature07655.

Hyde, W. T., T. J. Crowley, K. Y. Kim, and G. R. North (1989), A comparison of GCM and energy balance model simulations of seasonal temperature changes over the past 18,000 years, *J. Clim.*, *2*(8), 864–887, doi:10.1175/1520-0442(1989)002<0864:COGAEB>2.0.CO;2.

Hyde, W. T., T. J. Crowley, S. K. Baum, and W. R. Peltier (2000), Neoproterozoic "Snowball Earth" simulations with a coupled climate/ice-sheet model, *Nature*, *405*, 425–429, doi:10.1038/35013005.

Hyde, W. T., T. J. Crowley, S. K. Baum, and W. R. Peltier (2001), Reply to "Geophysics: Life, geology and snowball Earth" by D. P. Schrag and P. F. Hoffman, *Nature*, *409*, 306, doi:10.1038/35053172.

Kirschvink, J. L. (1992), Late Proterozoic low latitude glaciation: The Snowball Earth, in *The Proterozoic Biosphere: A Multi-disciplinary Study*, edited by J. W. Schopf and C. Klein, pp. 51–52, Cambridge Univ. Press, New York.

Leather, J., P. A. Allen, M. D. Brasier, and A. Cozzi (2002), Neoproterozoic snowball Earth under scrutiny: Evidence from the Fiq glaciation of Oman, *Geology*, *30*(10), 891–894, doi:10.1130/0091-7613(2002)030<0891:NSEUSE>2.0.CO;2.

Le Hir, G., G. Ramstein, Y. Donnadieu, and Y. Goddéris (2008), Scenario for the evolution of atmospheric pCO2 during a snowball Earth, *Geology*, *36*(1), 47–50, doi:10.1130/G24124A.1.

Li, Z. X., et al. (2008), Assembly, configuration, and break-up history of Rodinia: A synthesis, *Precambrian Res.*, *160*, 179–210, doi:10.1016/j.precamres.2007.04.021.

Love, G. D., et al. (2009), Fossil steroids record the appearance of Demospongiae during the Cryogenion period, *Nature*, *457*, 718–721, doi:10.1038/nature07673.

Macdonald, F. A., et al. (2010), Calibrating the Cryogenian, *Science*, *327*, 1241–1243, doi:10.1126/science.1183325.

McKay, C. P. (2000), Thickness of tropical ice and photosynthesis on a snowball Earth, *Geophys. Res. Lett.*, *27*(14), 2153–2156, doi:10.1029/2000GL008525.

Moczydlowska, M. (2008), The Ediacaran microbiota and the survival of Snowball Earth conditions, *Precambrian Res.*, *167*, 1–15.

Myhre, G., and F. Stordal (1997), Role of spatial and temporal variations in the computation of radiative forcing and GWP, *J. Geophys. Res.*, *102*(D10), 11,181–11,200, doi:10.1029/97JD00148.

Myhre, G., E. J. Highwood, K. P. Shine, and F. Stordal (1998), New estimates of radiative forcing due to well mixed greenhouse gases, *Geophys. Res. Lett.*, *25*(14), 2715–2718, doi:10.1029/98GL01908.

Nagy, R. M., S. M. Porter, C. M. Dehler, and Y. Shen (2009), Biotic turnover driven by eutrophication before the Sturtian low-latitude glaciation, *Nat. Geosci.*, *2*, 415–418, doi:10.1038/ngeo525.

North, G. R., R. F. Cahalan, and J. A. Coakley (1981), Energy balance climate models, *Rev. Geophys.*, *19*(1), 91–121, doi:10.1029/RG019i001p00091.

North, G. R., J. G. Mengel, and D. A. Short (1983), Simple energy balance model resolving the seasons and continents: Application to the astronomical theory of the ice ages, *J. Geophys. Res.*, *88*(C11), 6576–6586, doi:10.1029/JC088iC11p06576.

Olcott, A. N., A. L. Sessions, F. A. Corsetti, A. J. Kaufman, and T. F. de Oliviera (2005), Biomarker evidence for photosynthesis during Neoproterozoic glaciation, *Science*, *310*, 471–474, doi:10.1126/science.1115769.

Pari, G., and W. R. Peltier (1995), The heat-flow constraint on mantle tomography-based convection models: Towards a geodynamically self-consistent inference of mantle viscosity, *J. Geophys. Res.*, *100*(B7), 12,731–12,751, doi:10.1029/95JB01078.

Pari, G., and W. R. Peltier (1998), Global surface heat flux anomalies from seismic tomography-based models of mantle flow: Implications for mantle convection, *J. Geophys. Res.*, *103*(B10), 23,743–23,780, doi:10.1029/98JB01668.

Paterson, W. S. B. (1994), *The Physics of Glaciers*, 380 pp., Elsevier, New York.

Pavlov, A. A., O. B. Toon, A. K. Pavlov, J. Bally, and D. Pollard (2005), Passing through a giant molecular cloud: "Snowball" glaciations produced by interstellar dust, *Geophys. Res. Lett.*, *32*, L03705, doi:10.1029/2004GL021890.

Payne, A. J., et al. (2000), Results from the EISMINT model intercomparison: The effects of thermomechanical coupling, *J. Glaciol.*, *46*, 227–238, doi:10.3189/172756500781832891.

Peltier, W. R. (1998), Postglacial variations in the level of the sea: Implications for climate dynamics and solid-earth geophysics, *Rev. Geophys.*, *36*, 603–689, doi:10.1029/98RG02638.

Peltier, W. R. (2007), History of Earth rotation, *Treatise Geophys.*, *9*, 243–293, doi:10.1016/B978-044452748-6/00148-6.

Peltier, W. R., and Y. Liu (2008), Reply to comments by P. F. Hoffman et al. and Y. Goddéris and Y. Donnadieu on "Snowball Earth prevention by dissolved organic carbon remineralization," *Nature*, *456*(E9), doi:10.1038/nature07656.

Peltier, W. R., L. Tarasov, G. Vettoretti, and L. P. Solheim (2004), Climate dynamics in deep time: Modelling the "snowball bifurcation" and assessing the plausibility of its occurrence, in *The Extreme Proterozoic: Geology, Geochemistry and Climate*, *Geophys. Monogr. Ser.*, vol. 146, edited by G. S. Jenkins et al., pp. 107–124, AGU, Washington, D. C.

Peltier, W. R., Y. Liu, and J. W. Crowley (2007), Snowball Earth prevention by dissolved organic carbon remineralization, *Nature*, *450*, 813–818, doi:10.1038/nature06354.

Pierrehumbert, R. T. (2004), High levels of atmospheric carbon dioxide necessary for the termination of global glaciation, *Nature*, *429*, 646–649, doi:10.1038/nature02640.

Pierrehumbert, R. T. (2005), Climate dynamics of a hard snowball Earth, *J. Geophys. Res.*, *110*, D01111, doi:10.1029/2004JD005162.

Pollard, D., and J. F. Kasting (2005), Snowball Earth: A thin-ice solution with flowing sea glaciers, *J. Geophys. Res.*, *110*, C07010, doi:10.1029/2004JC002525.

Poulsen, C. J. (2003), Absence of a runaway ice-albedo feedback in the Neoproterozoic, *Geology*, *31*(6), 473–476, doi:10.1130/0091-7613(2003)031<0473:AOARIF>2.0.CO;2.

Poulsen, C. J., and R. L. Jacob (2004), Factors that inhibit snowball Earth simulation, *Paleoceanography*, *19*, PA4021, doi:10.1029/2004PA001056.

Ramanathan, V., M. S. Liou, and R. D. Cess (1979), Increased atmospheric CO_2: Zonal and seasonal estimates of the effect on the radiation energy balance and surface temperature, *J. Geophys. Res.*, *84*(C8), 4949–4958, doi:10.1029/JC084iC08p04949.

Raub, T. D. (2008), Prolonged deglaciation of snowball Earth, Ph.D. thesis, 443 pp., Yale Univ., New Haven, Conn.

Rieu, R., P. A. Allen, M. Plötze, and T. Pettke (2007), Climatic cycles during a Neoproterozoic "snowball" glacial epoch, *Geology*, *35*(4), 299–302, doi:10.1130/G23400A.1.

Rothman, D. H., J. M. Hayes, and R. E. Summons (2003), Dynamics of the Neoproterozoic carbon cycle, *Proc. Natl. Acad. Sci. U. S. A.*, *100*, 8124–8129, doi:10.1073/pnas.0832439100.

Schrag, D. P., and P. F. Hoffman (2001), Geophysics: Life, geology and snowball Earth, *Nature*, *409*, 306, doi:10.1038/35053170.

Schrag, D. P., R. A. Berner, P. F. Hoffman, and G. P. Halverson (2002), On the initiation of a snowball Earth, *Geochem. Geophys. Geosyst.*, *3*(6), 1036, doi:10.1029/2001GC000219.

Sohl, L. E., N. Christie-Blick, and D. V. Kent (1999), Paleomagnetic polarity reversals in Marinoan (ca. 600 Ma) glacial deposits of Australia: Implications for the duration of low-latitude glaciation in Neoproterozoic time, *GSA Bull.*, *111*(8), 1120–1139.

Stein, C. A. (1995), Heat flow of the Earth, in *Global Earth Physics: A Handbook of Physical Constants*, *AGU Ref. Shelf*, vol. 1, edited by T. J. Ahrens, pp. 144–158, AGU, Washington, D. C.

Tarasov, L., and W. R. Peltier (1997), Terminating the 100 kyr ice age cycle, *J. Geophys. Res.*, *102*(D18), 21,665–21,693, doi:10.1029/97JD01766.

Tarasov, L., and W. R. Peltier (1999), Impact of thermomechanical ice sheet coupling on a model of the 100 kyr ice age cycle, *J. Geophys. Res.*, *104*(D8), 9517–9545, doi:10.1029/1998JD200120.

Tarasov, L., and W. R. Peltier (2002), Greenland glacial history and local geodynamic consequences, *Geophys. J. Int.*, *150*, 198–229, doi:10.1046/j.1365-246X.2002.01702.x.
Tarasov, L., and W. R. Peltier (2004), A geophysically constrained large ensemble analysis of the deglacial history of the North American ice-sheet complex, *Quat. Sci. Rev.*, *23*(3–4), 359–388, doi:10.1016/j.quascirev.2003.08.004.
Tauxe, L., and D. V. Kent (2004), A simplified statistical model for the geomagnetic field and the detection of shallow bias in paleomagnetic inclinations: Was the ancient magnetic field dipolar?, in *Timescales of the Paleomagnetic Field*, *Geophys. Monogr. Ser.*, vol. 145, edited by J. E. T. Channell et al., pp. 101–116, AGU, Washington, D. C.
Tauxe, L., K. P. Kodama, and D. V. Kent (2008), Testing corrections for paleomagnetic inclination error in sedimentary rocks: A comparative approach, *Phys. Earth Planet. Inter.*, *169*, 152–165, doi:10.1016/j.pepi.2008.05.006.
Trindade, R. I. F., and M. Macouin (2007), Palaeolatitude of glacial deposits and palaeogeography of Neoproterozoic ice ages, *C. R. Geosci.*, *339*, 200–211, doi:10.1016/j.crte.2007.02.006.
Warren, S. G., and R. E. Brandt (2006), Comment on "Snowball Earth: A thin-ice solution with flowing sea glaciers" by David Pollard and James F. Kasting, *J. Geophys. Res.*, *111*, C09016, doi:10.1029/2005JC003411.
Warren, S. G., R. E. Brandt, T. C. Grenfell, and C. P. McKay (2002), Snowball Earth: Ice thickness on the tropical ocean, *J. Geophys. Res.*, *107*(C10), 3167, doi:10.1029/2001JC001123.
Williams, G. E. (2008), Proterozoic (pre-Ediacaran) glaciation and the high obliquity, low-latitude ice, strong seasonality (HOLIST) hypothesis: Principles and tests, *Earth Sci. Rev.*, *87*, 61–93, doi:10.1016/j.earscirev.2007.11.002.

Y. Liu and W. R. Peltier, Department of Physics, University of Toronto, 60 St. George St., Toronto, ON M5S 1A7, Canada. (ygliu@atmosp.physics.utoronto.ca)